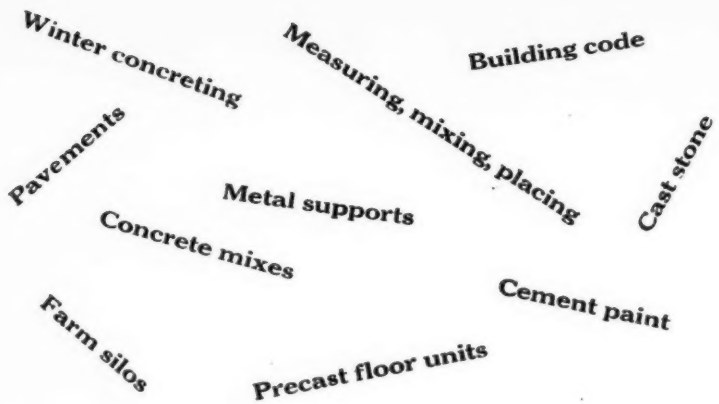
ACI News Letter

Vol. 22 No. 2 JOURNAL of the AMERICAN CONCRETE INSTITUTE October 1950



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AMERICAN CONCRETE INSTITUTE 18263 W. McNichols Rd. Detroit 19, Mich.

ACI News Letter Contents

October 1950

On the Cover—The dominant architectural interest of the new Portland Teamster building, Portland, Ore., is the main entrance featuring a background of vertical corrugated concrete upon which are set ribbon type aluminum letters and a precast sculptured bas-relief of the Teamster's emblem. The corrugated panel weighs about one ton and was cast of especially fine aggregate concrete after molding in clay.

Courtesy M. H. Hartford, Architect

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Thomson nominated for president

Kennedy receives vice-presidency nomination

The 1950 ACI Nominating Committee, consisting of Harmer E. Davis, chairman, J. F. Barbee, Robert F. Blanks, Herbert J. Gilkey, G. L. Lindsay, Harmon S. Meissner, Stanton Walker and Kenneth B. Woods, reports the following nominations for election at the 47th annual convention in San Francisco:

For President

Harry F. Thomson to succeed Frank H. Jackson for a one-year term beginning at the February 1951 convention.

Mr. Thomson, Manager, Redi-Mix Concrete Division, Material Service Corp., Chicago, Ill., has been prominently connected with the ready-mixed concrete industry for many years, and has been active in ACI affairs since 1928. He was elected to the ACI Board of Direction in 1939, serving until 1940. He was Director-at-Large, 1942-47, Director, 1948, and elected Vice-President in 1949. He was a member of the Publications Committee, 1940-41, and has served on numerous other ACI committees; he is the author of several JOURNAL papers as well as articles in other technical publications.

After graduating from Washington University in 1910, Mr. Thomson served as instructor and research worker at the Massachusetts Institute of Technology for 4 years. He then became associated with the Provident Chemical Works, successively as assistant superintendent, Eastern manager in New York City, sales manager, and (from 1920-24) vice-president. In 1924 he joined the Federal Phosphorous Co. as executive assistant.

In 1927 Mr. Thomson became vice-president of the General Material Corp. of St. Louis, one of the pioneer companies in producing commercial ready-mixed concrete. He continued with that company as vice-president and subsequently as president until

1948 when he was retained by Material Service Corp. to be in over-all charge of their extensive ready-mixed concrete service in the Chicago area.

Mr. Thomson has served as president of the National Ready Mixed Concrete Assn. and is a former national director of A.S.C.E.

For Vice-President

Henry L. Kennedy for 2-year term, beginning at the February 1951 convention.

The author of several papers in the ACI JOURNAL, Mr. Kennedy has been associated with the Institute since 1934. He is a member of Committee 115, Research, and Committee 613, Recommended Practice for the Design of Concrete Mixes; he has also been active in administrative work, being a member of the Board of Direction, 1944-49.

He is Manager of the Cement Division, Dewey and Almy Chemical Co., Cambridge, Mass., and as a consulting engineer he has specialized in reinforced concrete design and construction in the United States, Canada, Germany and England. Mr. Kennedy was for 10 years head of the construction course at Wentworth Institute, specializing in concrete instruction, and helped organize and direct the course in concrete technology at the Harvard Graduate School of Engineering.

A. T. Goldbeck, Engineering Director, National Crushed Stone Assn., Washington, D. C., elected for a 2-year term as vice-president at the 1950 convention in Chicago continues in that office.

Directors

For Directors, 3-year terms, four nominations for as many places on the Board: FRANK KEREKES, G. L. LINDSAY, I. E. MORRIS and WALTER H. PRICE.

Dean KEREKES, Assistant Dean of Engineering, Iowa State College, Ames, Ia., a member of TAC since 1948, is also active on Committee 315, Detailing Reinforced Concrete Structures, and 318, Standard Building Code. An ACI Member since 1938, he is also active in many other engineering and educational organizations besides being chairman of the Iowa Building Code Council.

Mr. LINDSAY, Director of Tests and Research, Universal Atlas Cement Co., New York City, has been an ACI Member since 1929. Upon graduation from the University of Minnesota in chemical engineering in 1921 he joined the company as a chemist at Duluth and in 1925 moved to the office at Chicago where he successively served as engineer, inspection bureau, and engineer of tests. He was assistant director of tests and research and was appointed to his present position in 1944.

Mr. MORRIS, a member of Committee 318, Standard Building Code, has been an ACI Member since 1947, and is a consulting engineer in Atlanta, Ga. For 8 years he was chief engineer of the reinforcing steel department of Connors Steel Co., Birmingham, Ala. In 1938 he entered private structural engineering practice and in 1944 organized the firm of I. E. Morris and Associates, engineers and consultants. Mr. Morris is a member of A.S.C.E., N.S.P.E. and the Georiga Engineering Society.

Mr. PRICE, Head of Materials Laboratory, U. S. Bureau of Reclamation, Denver, Colo., is chairman of two ACI committees—Committee 210, Resistance to Erosion in Hydraulic Structures, Committee 613, Recommended Practice for the Design of Concrete Mixes. An ACI Member since 1937, he is also the author of numerous JOURNAL papers. He has been with the Bureau since 1930, the past 15 years in materials testing and research on large concrete structures.

Nominating Committee

The 20 candidates for membership on the 1951 Nominating Committee—five to be elected—are:

WALDO G. BOWMAN	W. C. HANNA
ROY W. CARLSON	FRED HUBBARD
S. J. CHAMBERLIN	J. W. KELLY
MILES N. CLAIR	WILLIAM LERCH
H. F. CLEMMER	F. V. REAGEL
HERBERT K. COOK	T. E. STANTON
ROY W. CRÚM	BYRAM W. STEELE
RAYMOND E. DAVIS	A. G. TIMMS
JOSEPH DI STASIO	BAILEY TREMPER
JOHN R. DWYER	CHARLES S. WHITNEY

So you're going to San Francisco?

San Francisco-Oakland Bay Bridge

San Francisco, depicted as one of the few "skyline" cities of the nation, is situated on the slopes of numerous rolling hills at the tip of a peninsula bounded on three sides by the waters of the San Francisco Bay, the Golden Gate and the Pacific Ocean.

As seen from the air, San Francisco is the center of a great pendant with one thread leading eastward over San Francisco Bay, the other northward high above the Golden Gate. These connecting threads are actually mammoth bridges—the world's two largest—tying together the San Francisco Peninsula with the East Bay area in one direction and Marin Peninsula in the other.

The San Francisco-Oakland Bay Bridge, the world's longest and a link in U. S. Highways No. 40 and 50, is a monument to combined engineering genius and mechanical skill.

The bridge project from the San Francisco terminal to the Oakland terminal, end to end of approaches, is 8¼ mi. The bridge proper, including island crossing, is approximately 4½ mi. long and spans approximately 12,000 ft of navigable water.

The structure over the west channel is made up of two complete suspension bridges, with a central anchorage about in the middle of the west channel. The two main spans of the center suspension are 2310 ft each and the side spans 1160 ft. Vertical clearance ranges between 200 and 218 ft above the surface of low water. The towers of the suspension bridge are 474 and 519 ft above water.

The largest vehicular tunnel in the world, through Yerba Buena Island, connects the structures over the west and east channels. The bore is 76 ft wide, 58 ft high and 540 ft long. The structure over the east channel consists of one main cantilever span of 1400 ft with two 510-ft anchor arms. East of the cantilever span there are five truss spans, each 509 ft long.

The foundations of the bridge extend to the greatest depth below water of any ever before constructed by man, one pier being sunk 235 ft below water, another about 200 ft, and several others ranging from 105 to 180 ft.

Cables are 28¾ in. in diameter, containing 17,464 wires—a total of 70,815 miles, or enough to encircle the earth nearly three times.

The concrete and reinforcing steel in the bridge is sufficient to rebuild all the large office buildings of downtown San Francisco. The timber, 30,000,000 bd ft, would build 3000 five-room dwellings or the residential section of a town of 15,000 population. Each individual tower of the bridge, standing more than 700 ft high from base of pier on the floor of the bay to its top, represents a construction job comparable to a 60-story skyscraper.

If you haven't seen it before, the San Francisco-Oakland Bay Bridge is a must on your convention itinerary.

WHAT "Concrete Nuts"?

It came in an interval of transition between scheduled ACI program items—chair scraping, posture shifting, applause doing its perfunctory best to put the speaker, too, at *his* ease. The much loved and highly respected ACI'er eased himself graciously into comfortable casualness; made quick work of flinging out a short bridge to span the well-known chasm dividing speaker and listeners—three words: "we concrete nuts"; nothing malicious, no pointing fingers of accusation; nothing, I am sure, but fraternal warmth.

Such was a minor incident bolstering the old thought that ACI redefine itself *as a fraternity of interests* "going our way." On many occasions these interests are virtually parallel; at others, divergent in special directions. Yet from most of these divergencies come definite contributions fitting a broadly charted course.

Here and there a "nut" appears, perhaps from a too concentrated specialization. He rides his hobby so hard that he sees other riders only as blurred images.

The implications of these thoughts are pointed up in staff minds from at least two major and from many minor projects.

> Where is this Institute going? The question is asked here as freshly as though we hadn't been "going" for 45 years; as though we had not stated our objects in our Charter. What is to fix those objectives?

What is to fix them, indeed, except such a synthesis of least common denominators of interest and curiosity as thrive among more than 5000 individuals who wish to make knowledge their own keys to the solutions of their individual problems.

We know that with some preliminary assists from the original entrepreneurs of organization, ACI came into being in response to a need for more and more information about concrete as a means of SOLVING PROBLEMS of CONSTRUCTION; to that all lanes of thought lead. The National Association of Cement Users might almost be said to have occurred spontaneously 45 years ago from that need.

Questions became a bit more articulate and more differentiated by the time NACU became ACI. *Who* wanted to know *what*, and *why?* That differentiation is now more acute.

Through the Institute's membership classification project, ACI may adjust its direction. Classification, after several years of labor, is at a stage nearing emergence, significantly we hope. It should help to tell us *what* individuals, agencies, professions, industrial and other allies in construction activities, are finding it advantageous to be identified with ACI as a source of *what dues-payers think they want.*

Incidentally, but very critically as budget time approaches, the question arises: what apportionment of dues and other assists will best serve and satisfy the membership. Dancers pay fiddlers.

The classification (so far more than 80 percent of ACI's Members in continental United States) is shaping up, indicating major and minor activities which prompt the questions laid at our door.

> First to emerge is the picture of a large segment (devoted to the perfection of concrete) of a *very* large industry. Our segment embraces those who are important parts of the business of concrete construction; not only practice of design and the techniques of design inquiry, but the suppliers of materials, tools and equipment and men and brains with capacity to organize these as working forces of the CONSTRUCTION INDUSTRY.

In the broad strokes of a general view much detail will be obscure. A hundred specializations of fact and action are enveloped in the picture: those who direct and those who accomplish the tasks of testing, those who tell us what materials will "take" in punishment and by the same values what they will "give" in service to civilization.

The emerging ACI "census" of its members by jobs, by interests, by the very allegiances of employment, will not clearly show in group tabulation what we do clearly *see*:

> There is a dual existence of *competition* and *cooperation* in both knowing and doing, among students and among their instructors, among the institutions which give them haven; among the constructors who file competitive bids and develop and use the techniques which are especially theirs, and among the designers who work out the drawings on which bids are taken; among the producers of tools, equipment and materials; among highly specialized and contributing thought down the winding lanes of basic science and its practice—those called nuts sometimes win blue ribbons on eccentricities; and that, too, both within the Institute and between the Institute and other organizations going different but similar ways there are like forces of health in cooperation and competition toward the evolution of structures to serve us in living and in working toward better ways.

Harrywhipple

New Members

ACI—like the "horseless carriage"—is here to stay

Evidently one new member has put a lot of thought into joining ACI, but finally decided that the Institute was here to stay. Institute headquarters recently received from Boston, Mass., a membership application form printed in 1928.

This doesn't mean Bostonians are slow in getting things done—some of ACI's most active members reside there. We're glad to welcome the new member and assure him that ACI is indeed here to stay and still growing.

(Total of new members in August was 47, so evidently others also agree with the Boston member.)

Alabama

McLemore, Richard Henry, (Indiv.) 536 S. Lawrence St., Montgomery, Ala.

Arizona

Jobusch, Fred Henry, (Indiv.) 650 N. Sixth Ave., Tucson, Ariz.

Arkansas

Bissett, James Robert, (Indiv.) 314 Engineering Bldg., University of Arkansas, Fayetteville, Ark.

California

Bailly, Florent H., (Indiv.) 709 S. Fair Oaks Ave., Pasadena 2, Calif.

Bresler, B., (Indiv.) 303 Engineering Materials Lab., University of California, Berkeley 4, Calif.

Eldridge, Robert A., (Indiv.) 1709 Oak Ave., Manhattan Beach, Calif.

Moran, Donald F., (Indiv.) Pacific Fire Rating Bureau, 548 S. Spring St., Los Angeles 13, Calif.

Olsen, Homer J., (Jr.) 1018 W. Cypress St., San Dimas, Calif.

Replogle, William H., (Indiv.) 1732 5th Ave., Apt. No. 4, San Diego 1, Calif.

Colorado

Bliss, Major G., (Indiv.) 2590 S. Fillmore, Denver 10, Colo.

Gilliland, John L. Jr., (Indiv.) Bureau of Reclamation, Denver Federal Center, Denver, Colo.

District of Columbia

Finglass, William (Indiv.) c/o American Construction Co., Inc., 261 Constitution Ave., N. W., Washington 1, D. C.

Florida

Shartle, J. Herbert, (Indiv.) 4413 S. W. 13 Ter., Miami 34, Fla.

Idaho

Linton, Donald, (Indiv.) RFD No. 2, Pocatello, Idaho

Illinois

Ball, Arthur L. Jr., (Jr.) Box 498, Libertyville, Ill.

Bykowsky, Nicholas, (St.) 12217 S. Normal Ave., Chicago 28, Ill.

Indiana

Horne, E. S., (Indiv.) Ready Mixed Concrete Corp., 1100 Burdeal Parkway Blvd., Indianapolis 8, Ind.

Trutschel, Henry, (St.) 301A Victory Lane, Angola, Ind.

Michigan

Mazneff, Karl K., (Indiv.) 16877 Stout, Detroit 19, Mich.

Ranney, Charles C., (Indiv.) 1305 Bronx Ave., Kalamazoo, Mich.

Minnesota

Hill, Hibbert, (Indiv.) c/o Northern States Power Co., 15 S. 5th St., Minneapolis 2, Minn.

Mississippi

Marable, James Rose, (Jr.) 1905 Clay St., Vicksburg, Miss.

New York

Kirchner, Norman F., (St.) 29 Wohlers Ave., Buffalo 8, N. Y.

Lancaster, G. Graham, (Indiv.) Orinoco Mining Co., Room 1401, 25 Broad St., New York, N. Y.

Lepaluk, Miroslaus, (Indiv.) 254 E. 3rd St., New York 9, N. Y.

Packer, George, (Indiv.) 31-63 30th St., Long Island City 6, N. Y.

Ohio

Steinmann, William N., (Indiv.) Route No. 5, Box 42-M, Poage Farm Road, Cincinnati 15, Ohio

Tennessee

Stottlemyer, Paul Clinton, (Indiv.) Room 415, Union Bldg., Tenn. Valley Authority, Knoxville, Tenn.

Texas

Harris, Robert S., (St.) 1408A Brackenridge Apts., Austin, Texas

Sutton, James L., (St.) Unit 281, Oak Grove Cts., Austin 5, Texas

Virginia

McGaughy, John B., (Indiv.) 700 Front St., Norfolk 7, Va.

Continued on p. 23

Darex AEA

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Darex AEA

is being used in all types of concrete work, all over the world.
Distribution warehouse stocks are conveniently located throughout North and South America and in most foreign countries.

The Conemaugh Dam near Pittsburgh is part of the Ohio River Basin flood control program. Darex AEA is used in the 1,090 ft. concrete structure to improve workability, increase resistance to freezing and thawing. *Photo courtesy U. S. Army Corps of Engineers, Pittsburgh District.*

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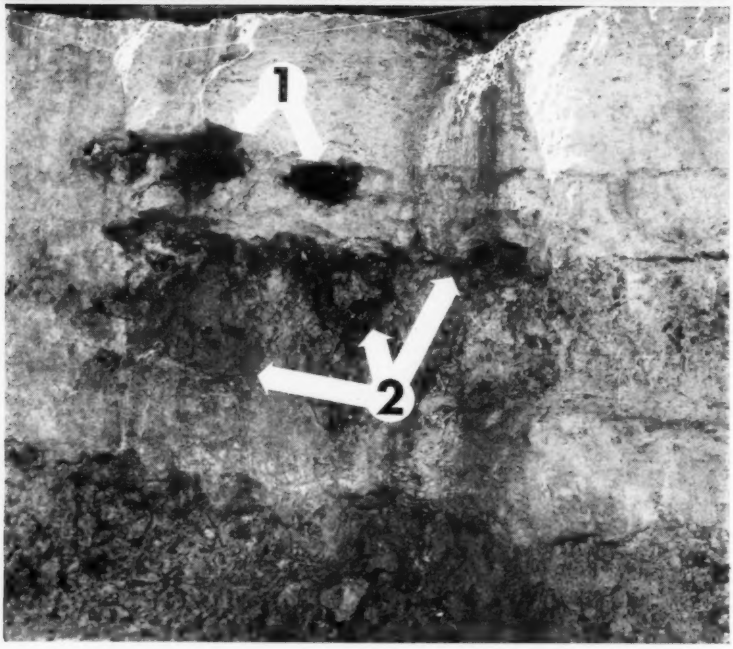

Pictures Used through Courtesy U. S. Corps of Engineers.

The rock wall illustrated above, exposed by excavation for the navigation lock, shows the penetration and distribution of Intrusion Grout throughout the Tampa limestone substructure. The black coloring matter used in the grout is easily recognized in this unretouched photo.

1. Large cavities and solution channels filled solidly with Intrusion Grout which has penetrated all irregularities and solidly plugged these voids.

2. Lateral cracks and solution channels are completely filled and effectively prevent inflow of water.

Pictures tell better than words the success of the Intrusion sand-grouting operation at Jim Woodruff Dam, Chattahoochee, Florida.

The difficult unwatering problem in excavating for the navigation lock was solved by a solid grout curtain placed in the porous Tampa limestone under the cofferdam. This curtain reduced inflow of water into the excavation to a point where it

THAN CEMENT!

could be economically controlled by pumping.

The excavation for the navigation locks was located within 300 feet of the river bank and had a depth of 30 feet below the normal river level.

Exploratory holes, pumping, and other tests proved that dewatering of the excavation by pumping would be extremely costly.

The grout curtain was installed by drilling 2½" grout holes approximately 15 feet deep and 10 feet apart (in lineal fashion) across the effective areas, then pumping Intrustion Grout down into the rock substructure under 1 p.s.i. per foot of depth. The Intrusion Grout mixture consisted of (1) water, (2) Intrusion Aid, (3) Alfesil, (4) cement, (5) sand, and (6) lampblack coloring for easy identification of grout. As distinguished from neat grout, the sand mixtures used *ran as high as ten cubic feet of sand per bag of Portland cement.*

The size of this grouting operation is indicated by the fact that 152,020 cu. ft. of Intrusion Grout were placed in 2,824 lin. ft. of grout curtain.

The inflow of water under the cofferdam was effectively checked in spite of river level increases as high as 10 ft.

The cost of the entire project was considerably lowered by the use of Intrusion Grout and Intrusion methods, the greater part of this savings being made possible by the low cement and high sand percentages used in the Intrusion Grout.

Intrusion-Prepakt engineers will be glad to show you, without cost or obligation, the advantages of Intrusion-Prepakt methods and materials as applied to your specific problem.

Showing part of grouting procedure —setting grout riser through a hole puncher.

General view of lock excavation, showing how effectively grout curtain has stopped inflow of water.

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Positions and Projects — ACI Members

Concrete chimney specifications

The report of ACI Committee 505, Reinforced Concrete Chimneys, "Tentative Specification for the Design and Construction of Reinforced Concrete Chimneys (505-36T)," has been reprinted by the Institute due to the large demand for an authoritative reference on such construction. The 44-page report was adopted as a tentative standard (505-36T) at the Institute's 32nd Annual Convention in Chicago, Feb. 25, 1936. The report is available from ACI headquarters at $1.50 per copy, $1.00 to ACI Members.

Miles Clair named president of Thompson and Lichtner Co.

Miles N. Clair, nationally known in concrete engineering and active in ACI technical and administrative committees, has been named president of the Thompson and Lichtner Co., Inc., Brookline, Mass. He has been serving as vice-president of the organization.

Brouk named president of Perlite institute

J. John Brouk, Precast Slab and Tile Co., St. Louis, Mo., was elected president of the Perlite Institute at its September meeting. Wharton Clay, also an ACI Member, serves as secretary and treasurer of the organization. Recommendations were adopted at the meeting for the placing of perlite cement concrete with or without air-entraining agents.

Perry opens office

G. Nelson Perry, formerly chief structural engineer with Thomas Worcester, Inc., Boston, Mass., has established the structural engineering office of G. Nelson Perry & Co. in Boston. Mr. Perry is president of the company. He graduated in 1929 in civil engineering from Northeastern University and joined the J. R. Worcester and Co. consulting organization as a draftsman and was its senior designer when the firm broke up in 1946. He then became associated with the Thomas Worcester staff.

"Stone, steel and dreams"

"Bridges are made of stone, steel and dreams," stated D. B. Steinman, noted bridge engineer of New York City, at the September meeting of the Detroit chapter of the Michigan Society of Professional Engineers.

Dr. Steinman, an ACI Member since 1927, presented a colorful portrayal of the engineer's creative ability to combine science and art in building great and beautiful spans. He illustrated his talk, "The Romance of Bridges," with slides of many famous bridges and showed how the engineer should not only make a bridge structurally sound, but should also design it pleasing to the eye and fit the bridge into the surrounding landscape.

He described vividly some of the drama and tragedy that has attended the construction of several world-famous bridges, but also focused attention on the many successful projects, both in beauty and strength.

He closed by saying that the engineer, by combining science and art in the construction of a bridge, leaves a useful, living monument for the future.

Schwarz retires

Walter A. Schwarz, formerly resident engineer at Dorena Dam, a unit of the Wilamette Basin Project in western Oregon, retired from the Corps of Engineers recently after 16 years of government service. Before joining the Corps of Engineers he was a district engineer with the Washington State Highway Department, camp superintendent for the forest service, and director of operations for WPA in Washington State.

Arnold to study German precast industries

Mack A. Arnold, president of Arnold Stone Co., Inc., Greensboro, N. C., has accepted the post of technical consultant in the Industry Division of the United States High Commissioner for Germany at Frankfurt. He is an expert on precast concrete construction and was sent to Germany by the U. S. Department of Commerce to investigate new types of concrete developed by the Germans during World War II.

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Maximum Bond

WITH THIS BAR

When you use the improved Bethlehem Reinforcing Bar, you can be sure of maximum bond strength for the life of the structure.

The Bethlehem Reinforcing Bar provides maximum bond strength because of its high, closely-spaced lugs, as called for in ASTM Specification A305-49. The anchorage these lugs make possible means low slips at working loads. This helps to prevent the formation of wide tensile cracks in the concrete, and results in better-looking, more economical structures.

The Bethlehem Reinforcing Bar is made from new-billet steel in sizes from ⅜ in. to 1¼ in. It conforms in every way to the requirements of ASTM Specification A305-49. To be sure of maximum bond, specify it for your next concreting job.

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On the Pacific Coast Bethlehem products are sold by Bethlehem Pacific Coast Steel Corporation.

Export Distributor: Bethlehem Steel Export Corporation

The size of each Bethlehem Reinforcing Bar is readily determined by means of an identifying numeral rolled into the bar. In this picture, No. 8 denotes a 1-in. round.

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Topractsoglou joins University of Texas staff

A. A. Topractsoglou, formerly at Lehigh University, has accepted the position of assistant professor in the Civil Engineering Dept. at the University of Texas, Austin, Texas.

California road conference scheduled

California's Conference on Street and Highway Problems, an annual event for road men from every level of government and private enterprise concerned with highways, roads and streets, has been scheduled for Jan. 24-26, 1951. The meeting will be held on the Berkeley campus of the University of California. First organized in 1949, the conference is presented each year by the Institute of Transportation and Traffic Engineering of the University of California. Chairman for the 1951 meeting will be Ralph G. Wadsworth, City Engineer of San Francisco.

International Council for Building Documentation

The first General Assembly of the International Council for Building Documentation will meet in Paris, France, Oct. 23-31. The building documentation council, organized in October 1949 by the United Nations Economic Commission for Europe, will examine the problems concerning building documentation and the circulation of information among engineers, architects, public administrators, building organizations, etc., of member countries of the United Nations.

The aim of the council is to better the information and documentation offered to building technicians throughout the world, thus facilitating efficient international cooperation between the various national organizations concerned with the improvement of building methods or their implementation.

Building Research Congress to feature papers on influence of research on structural design

The Building Research Congress to be held in London, England, Sept. 11-20, 1951, during the Festival of Britain, will feature several papers dealing with the influence of modern research on concrete structural design. There will be an introductory paper on modern developments in reinforced concrete design and how it has been influenced by research; a review of progress in the field of research on reinforced concrete will be the topic of another paper.

Research on and development of prestressed concrete will be discussed and a paper on problems in the design and use of thin concrete shells will be presented.

The Congress will also feature several papers dealing with concrete quality control and the test methods applied, accelerated curing methods and high early strength concrete. An omnibus review of developments in European and American building materials will include lightweight concrete made with lightweight aggregates and aerating agents.

The Building Research Congress is sponsored by the British professional institutions, technical societies and governmental departments, in cooperation with representative industrial federations.

Danish National Institute of Building Research

In the Danish National Institute of Building Research, Copenhagen, Denmark, second annual report, research activities in winter concreting, building standardization, sound and heat insulation and 17 other projects are reported. The Institute was established in 1947 to promote and coordinate technical and economic research which may contribute to improvement and reduced costs of building.

To test the practicality of recommended practices for winter concreting suggested by the Danish institute, it was arranged with contractors on 42 projects of varying nature, dimensions and stages throughout Denmark to follow such practices; reports were kept on all operations, problems encountered and costs. Another project completed was a study of seven Danish concrete mixers. Also of interest was the research conducted on

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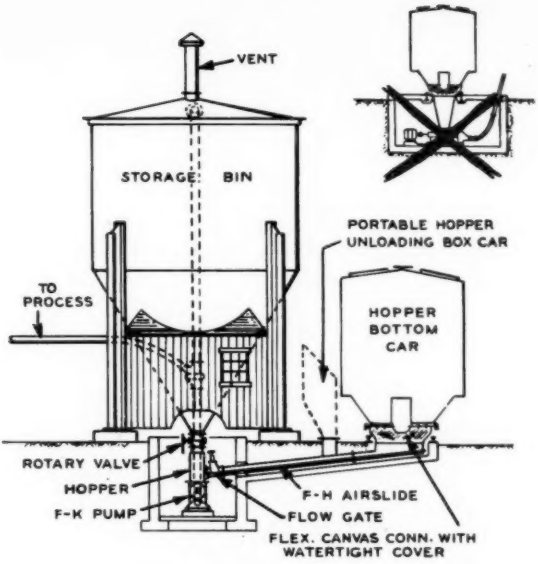

FULLER-KINYON PUMP with F-H AIRSLIDE eliminates pit under track

No longer is it necessary to excavate and build a concrete pit under the railroad track to use a Fuller-Kinyon Pump for unloading bulk Portland cement from hopper-bottom cars. This expense and labor has been eliminated through the use of the F-H Airslide Conveyor, an ingenious device for conveying with low-pressure air. Materials can also be unloaded in this manner from box cars by using a portable hopper.

The drawing above clearly illustrates the use of the Fuller-Kinyon Pump and the F-H Airslide Conveyor in such an installation. The F-H Airslide conveys materials from the car to the pump, underneath the bin, the pump being used to convey to the storage bin, or if desired, direct to process in the plant. The same pump will reclaim from storage bin for delivery of materials to process.

Fuller-Kinyon Pumps and F-H Airslides . . . the answer to many a conveying problem. May we suggest that you investigate these modern systems for conveying dry pulverized materials. Minimum expenditure on your part may result in major savings.

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San Francisco 4 • 420 Chancery Bldg.

DRY MATERIAL CONVEYING SYSTEMS
AND COOLERS—COMPRESSORS AND
VACUUM PUMPS—FEEDERS,
AND ASSOCIATED EQUIPMENT

fuller

P-120

Who's Who This Month

ACI Committee 617

ACI Committee 617, Specifications and Recommended Practice for Concrete Pavements and Bases, under the chairmanship of H. F. Clemmer, Materials Engineer, Government of the District of Columbia, Washington, proposes a revised and amended specification, "Proposed Revision of Specifications for Concrete Pavements and Bases (617-44)," p. 93.

This standard, if adopted at the 47th annual ACI convention in San Francisco in February, would replace ACI Standard 617-44. New specifications incorporated in the proposed standard include those covering air entrainment, removal of forms, premolded joint fillers and joint filling materials, and method of placing reinforcement. The section under soil foundation preparation has also been revised.

Other members of the committee are A. T. Goldbeck, National Crushed Stone Assn.; W. E. Hawkins, North Carolina State Highway Dept.; Fred Hubbard, National Slag Assn.; Frank H. Jackson, Bureau of Public Roads; Ted J. Kauer, Ohio Dept. of Highways; Bailey Tremper, Washington State Dept. of Highways; Stanton Walker, National Ready Mixed Concrete Assn.; and M. Allan Wilson, Delaware State Highway Dept.

L. S. Brown and C. U. Pierson

A "Linear Traverse Technique for Measurement of Air in Hardened Concrete" is described on p. 117 by L. S. Brown, Senior Research Petrographer, Portland Cement Assn., Chicago, Ill., and C. U. Pierson, Chief Research Assistant, Technical Dept., Marquette Cement Manufacturing Co., Chicago, Ill.

Mr. Brown was author of Chapter IV, "Long-Time Study of Cement Performance in Concrete," May 1948 JOURNAL. He has been geologist and petrographer for the Portland Cement Assn. since 1940. He graduated in chemistry from New Mexico State College in 1916 and received his M.S. in petrography from the University of Oklahoma in 1928 and Ph.D. from the Massachusetts Institute of Technology in 1933. Before he joined the Portland Cement Assn. he was with the Lone Star Cement Corp., 1935-40.

A graduate in engineering geology from the University of California, Mr. Pierson worked for the Shell Development Co. from 1937-42 in chemical research. After 4 years of service with the Army ordnance department, he returned to Shell in 1946, joining the staff of the Portland Cement Assn. as petrographer in the Field Research Bureau in 1948. While with the association, his work consisted of microscopic examination of aggregates, concrete, and studies relative to air entrainment in hardened concrete. He recently joined the Marquette Cement Manufacturing Co. as Chief Research Assistant.

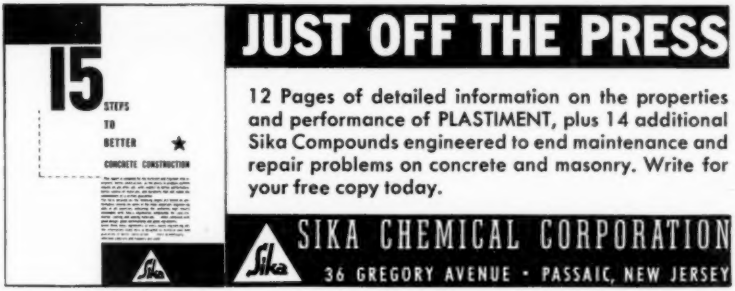
Rachel Friedland

A newcomer to JOURNAL pages, Rachel Friedland, Assistant Professor of Building Materials, Hebrew Institute of Technology, Haifa, Israel, is the author of "Influence of the Quality of Mortar and Concrete upon Corrosion of Reinforcement," p. 125.

Professor Friedland graduated from the Hebrew Institute of Technology in 1931 in civil engineering and has been a member of the faculty since then. She served as Head of the Dept. of Standard Testing (Building Materials) of the Institute from 1940 to 1948 and has been chairman of several committees for Israel standards. The author of a book on cement and concrete and numerous papers in Hebrew technical journals, Professor Friedland also held the rank of major in the Israel army, 1948-49.

Thomas B. Kennedy

"Proper Sand Grading Improves Mass Concrete," p. 141, is the second JOURNAL contribution by Thomas B. Kennedy, Chief, Physical Test Branch, Concrete Research Division, Corps of Engineers, Waterways Experiment Station, Jackson, Miss. His other paper, in the Nov. 1944 JOURNAL, discussed a limited investigation of capping materials for concrete specimens.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He is a member of ACI Committee 613, Recommended Practice for the Design of Concrete Mixes, and Committee 115, Research. Mr. Kennedy graduated in chemistry from the University of Illinois in 1930 and since that time has worked mainly in the concrete field for the Corps of Engineers. He was actively engaged in construction work for several years and has been associated with the Concrete Research Division at the Waterways Experiment Station for the past 11 years.

J. L. Gilliland and T. R. Bartley

"Water-Solubility of Alkalies in Portland Cement," p. 153, is authored by J. L. Gilliland and T. R. Bartley, both of the Research and Geology Division of the U. S. Bureau of Reclamation, Denver, Colo. It is the first appearance of either of these men in the JOURNAL.

Mr. Gilliland has been on the staff of the Bureau since 1933 and is now Head of the Chemical and Cement Laboratories. He has been closely associated with the alkali-aggregate problem since the days of the first indication at Parker Dam. A 1933 graduate in chemical engineering from the University of Colorado, Mr. Gilliland is a member of ACI, American Chemical Society and the National Assn. of Corrosion Engineers.

Mr. Bartley received his B.S. in chemistry from Indiana State Teachers College in 1940. Shortly thereafter, he was employed by Seagram's Distillery in Louisville, Ky., as a plant operator. From 1941-45 he served with the Medical Dept. of the Army. He has been a chemist in the Bureau's laboratory since 1946.

K. P. Billner

A new method of prestressing is described by K. P. Billner, President, Vacuum Concrete, Inc., Philadelphia, Pa., in "New Prestressing Method Utilizes Vacuum Process," p. 161. Well-known to JOURNAL readers, Mr. Billner's previous contributions have described electric prestressing of reinforcing steel (June 1943) and vacuum processes applied to precast housing units (Oct. 1949).

Awarded the Franklin Institute John P. Brown medal in 1947 for his work with vacuum processing of concrete, his name is also identified with such inventive contributions to concrete engineering as Aerocrete, the use of aluminum powder in concrete to improve bond to steel, vacuum concrete, interlocked helical reinforcement (sometimes referred to as "bed spring" reinforcing), electric prestressing, and the mechanical method of prestressing described in this paper.

Mr. Billner has been active in engineering and building construction in Sweden and the United States. He is the founder and president of Vacuum Concrete, Inc., and is a director of the Aerocrete Corp. of America and organized similar concerns in Canada and England. He is a member of ACI Committee 324, Precast Reinforced Concrete Structures.

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Honor Roll

February 1 to August 31, 1950

The Honor Roll is not a cold statistical list. Dorothy Hall, Editorial Secretary, has been doing a whale of a lot of research in ACI history. Interesting, this history, in how it **repeats**—and **doesn't.** She has been screening Honor Rolls for the period since they began. She is acquainted (long after the fact) with the member-getting propensities of ACI Members for 25 years. She can tell **who** have been good Institute proselyters in all those years—1, 5, 10 or more year "dashes" and in the 25-year marathon. To them the Institute owes it's healthy growth.

Early next year we'll tell how Duff Abrams won a very special award a long time ago.

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Continued on p. 20

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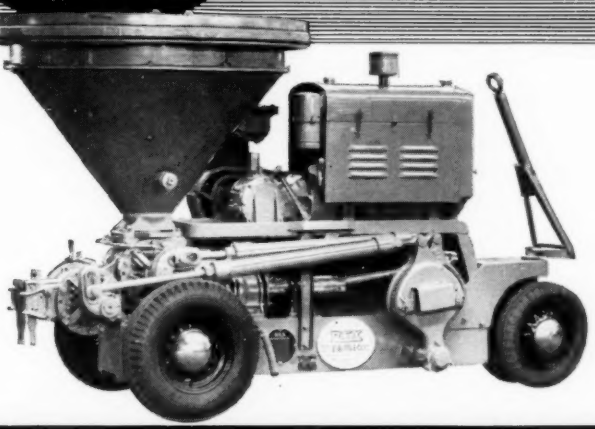

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Continued from p. 18

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Danish building research

Continued from p. 14

houses built of rammed stabilized earth, *geotek*. Compressive strengths between 10 and 30 kg per sq cm (142 and 427 psi) were obtained and with an addition of 6 percent cement to the earth, strengths of 45 to 75 kg per sq cm (640 to 1068 psi) were possible.

Hydro-Electric Power Commission of Ontario exceeds '48 construction peak

The construction of large power developments by the Hydro-Electric Power Commission of Ontario, Canada, in 1949 exceeded the all-time peak of construction activities reached in 1948 the Commission disclosed recently in its annual report. It also reported that in 1949 the Commission began the standardization of frequency in the southern Ontario system and the inspection of power transmission lines by helicopter.

At the end of 1949 there were several hundred separate jobs under construction, ranging from small projects costing one or two thousand dollars to power developments costing many millions of dollars. These bigger projects included the developments at Des Joachims, La Cave, Chenaux, Tunnel and Pine Portage.

The post-war construction and development program of the Commission, costing about $593,000,000, includes nine hydro-electric projects and two steam-electric stations. Since the war five of these projects have been placed in service.

Engineering is leading profession

The parallel growth of industry and the engineering profession has today reached the point where engineering is the leading profession for men, with more than 375,000 practitioners, and it is still the fastest growing, stated the June *Industrial Bulletin* of Arthur D. Little, Inc.

Approximately 51,000 men and women received degrees in engineering and science from more than 150 U. S. technical schools in June—a far cry from 1835, when Rensselaer Polytechnic Institute granted the first nonmilitary engineering degrees in the United States. West Point, opened in 1802, had granted engineering degrees, but its program stressed military engineering rather than the "civil" branches.

The early engineering schools had a difficult task in combatting the general public feeling that engineering education was inferior to classical studies. It was not until 1845 that the first School of Engineering was established as a branch of a classical college at Union College, and only two or three men were graduated each year until 1860.

In the United States, the growth in higher technical education coincided with the industrial revolution, but advanced technical schools had been founded in France and Germany somewhat earlier, the first in 1747.

After the Civil War, even though a substantial percentage of American engineers and chemists continued to be trained abroad, the increase in technical schools became one of the phenomena of American education. One factor in this growth was the Morrill Land Grant Act, which specified that the colleges so aided should teach "such branches of learning as are related to agriculture and the mechanic arts."

At a relatively early stage the engineering curriculum crystallized into its present pattern. As early as 1852 Rensselaer had developed the educational philosophy and curriculum which has become the pattern for modern colleges of engineering.

As the United States grew away from dependence on European products, and as commerce and industry became more competitive the need for trained engineers grew. The haphazard characteristics of early industrial efforts gave way to skillfully engineered enterprises, making engineering the leading profession of today and still growing.

"Snowed under" in October

Since publication of the *ACI 20-Year Index*, Institute headquarters has been "snowed under" trying to handle all the orders for copies of the *ACI Book of Standards*, the *ACI Inspection Manual*, and the *Detailing Manual*. Do you have these books on your reference shelf?

See the *20-Year Index* for a summary of the contents of each.

New Members

Continued from p. 8

Wilson, J. Gibson, Jr., (Indiv.) 4616 N. 19th Rd., Arlington 7, Va.

Washington

Johnson, Melvern E., (Indiv.) 2323 Third Ave., Seattle, Wash.

Whitacre, Horace J., (Indiv.) Consulting Engineer, 1401 Puget Sound Bank Bldg., Tacoma 2, Wash.

Wisconsin

Gebhard-Berghammer, Inc., (Corp.) 5420 W. State St., Milwaukee 8, Wis. (Jerome P. Gebhard)

Porto Rico

Rodriguez, Jose B., (Indiv.) No. 67 Matienzo Cintron St., Hato Rey, Porto Rico

Austria

Czernin, Wolfgang O., (Indiv.) Director of Research, Austrian Portland Cement Association, Wissgerberlande 44, Vienna III, Austria

Canada

Safir, Otto, (Indiv.) c/o Marwell Construction Co., Ltd., 410 Seymour St., Vancouver, B. C., Canada

Germany

Walz, Kurt, (Indiv.) Cannstatterstrasse 212, Stuttgart, Germany

Guatemala

Castillo Contoux, Mauricio, (St.) 6a Calle Poniente No. 84, Ciudad de Guatemala, Gautemala

India

Desai, D. S., (Indiv.) c/o Braithwaite & Co. (India) Ltd., P. O. Box 427, Calcutta, India

Iraq

Abdul-nour, Hazim Majid, (Indiv.) Hosh el Khan St., Mosml, Iraq

Philippine Islands

Erum, Emil N., (Indiv.) Scottish Rite Temple, 912 Taft Ave., Manila, P. I.

Galinato, Juan G., (Indiv.) 1226 Agno St., Malate, Manila, Philippines

Garrido, Jose, (Indiv.) 55 Alejandro Vi—Sanipaloc, Manila, P. I.

Scotland

McDougall, Archibald, (Indiv.) 19 the Bungalows, Affric Camp, Cannich, By Beauly, Inverness-shire, Scotland

Venezuela

Gonzelez-Gomez, C., (Indiv.) la Ave. Las Delicias No. 5, Caracas, Venezuela

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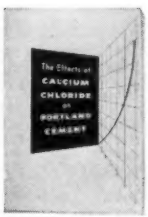

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SYNOPSES of recent ACI Papers and Reports

Institute papers of this JOURNAL Volume 22 are currently available at prices indicated. Please order by title and title number.

SOME OBSERVATIONS ON THE USE OF REINFORCING STEEL IN CONCRETE PAVEMENTS.....47-1

Price 35 cents.

BENGT F. FRIBERG—Sept. 1950, pp. 1-16 (V. 47)

The paper contains a few accumulated observations from a review of literature on the evolution of reinforced concrete pavement designs, with reference to various systems of reinforcement which have seen extended use. Typical structural concrete pavement failures are described. Trends in design and reinforcement practice are shown, with special reference to recent developments. Pertinent findings of a few published pavement surveys are cited, with special attention to the 900-mile Louisiana pavement survey made in 1945. The need for performance information and additional research is stressed.

TESTS OF PAPER MOLDS FOR CONCRETE CYLINDERS.....47-2

Price 35 cents.

ROBERT A. BURMEISTER—Sept. 1950, pp. 17-24 (V. 47)

Concrete test cylinders cast in a new type paper mold had a compressive strength lower than that predicted for the concrete mix used. Investigation showed that cracks and mechanical injuries to the outer shell of the concrete cylinder caused by movement of the paper stock during the first 24 hours of curing reduced the strength of the cylinder. To a lesser degree this was true also of the paraffined paper molds in common use for casting test cylinders, specimens cast in both types of paper molds showing lower strengths than test cylinders from the same mix cast in steel molds.

ADMIXTURES IN CONCRETE.....47-3

Price 60 cents.

W. T. MORAN, F. H. JACKSON, BRUCE E. FOSTER and T. C. POWERS—Sept. 1950, pp. 25-52 (V. 47)

Five papers by members of ACI Committee 212, Admixtures, are, because of their common general subject, presented together.

Various admixtures are discussed briefly as an introduction to more detailed treatment of air-entraining materials. The relative merits of admixtures and interground agents are considered. Optimum ranges of air content for different structural uses are given with particular reference to pavements.

The advantages and disadvantages of several types of admixtures used in the fabrication of various concrete products, such as building block, cast stone, pipe, cribbing and curbing, are discussed. The admixtures considered are classified into the following groups: accelerators, air-entraining agents, gas-forming agents, water repellent agents, and workability agents.

The factors affecting bleeding characteristics and workability of fresh concrete are reviewed and the effect of admixtures on these properties is assessed.

Present knowledge of admixtures in counteracting alkali-aggregate reactions is reviewed. It is emphasized that further studies may revise thinking in this field.

The effectiveness of various concrete admixtures in inhibiting the capillary flow of water and the flow of water under pressure is considered. The types of admixtures included in the discussion are accelerators, soaps, butyl stearate, finely subdivided dry materials, mineral oil, workability agents, and a miscellaneous group of proprietary compounds.

IMPROVED SONIC APPARATUS FOR DETERMINING THE DYNAMIC MODULUS OF CONCRETE SPECIMENS.....47-4

Price 35 cents.

C. E. GOODELL—Sept. 1950, pp. 53-60 (V. 47)

After a brief introduction to sonic testing and a description of commercial equipment, the apparatus built for the Michigan State Highway Department is discussed. Reliable results can be obtained by an unskilled worker with this compact equipment which has twice the frequency band spread of the usual oscillator. A wiring diagram of the sonic apparatus is included.

ANALYSIS OF THREE-DIMENSIONAL BEAM-AND-GIRDER FRAMING.47-5

Price 35 cents.

PHIL M. FERGUSON—Sept. 1950, pp. 61-72 (V. 47)

The beam-and-girder floor with some beams carried directly by columns and others supported on girders is cited as a practical problem in frame analysis that must include the torsional stiffness of the girder. Curves showing how moment coefficients vary with this torsional stiffness are developed for a few simple cases in interior panels. These show the weakness of rule-of-thumb methods.

A practical calculation form is set up for use with the moment distribution method in solving three-dimensional problems of this type.

Proposed Revision of SPECIFICATIONS FOR CONCRETE PAVEMENTS AND BASES (617-44).....47-6

Price 35 cents.

REPORT OF COMMITTEE 617—Oct. 1950, pp. 93-116 (V. 47)

New specifications incorporated include those covering air entrainment, removal of forms, premolded joint fillers and joint filling materials and method of placing reinforcement. Definitions have been added under soil foundation preparation and other parts of this section have been revised.

LINEAR TRAVERSE TECHNIQUE FOR MEASUREMENT OF AIR IN HARDENED CONCRETE.....47-7

Price 35 cents.

L. S. BROWN and C. U. PIERSON—Oct. 1950, pp. 117-124 (V. 47)

The method described for the determination of air in hardened concrete permits the examination of 6 x 8-in. and 6 x 10-in. random plane face-ground hardened concrete specimens which more truly represent the aggregate and air voids in the actual concrete than smaller specimens. The construction and use of the instruments are discussed and results of tests are given. Because of the time and equipment necessary to measure air content by means of the integrator, it is not adaptable to field use. However, as a laboratory tool it provides a means for quick and accurate determination of total air.

INFLUENCE OF THE QUALITY OF MORTAR AND CONCRETE UPON CORROSION OF REINFORCEMENT. 47-8

Price 35 cents.

RACHEL FRIEDLAND—Oct. 1950, pp. 125-140 (V. 47)

In tests to determine the influence of the quality of mortar and concrete upon corrosion of reinforcement the variables studied were cement content, water-cement ratio, consistency, grading and depth of cover. The specimens, stored in moist air or exposed to weather, were tested up to the age of 2 years.

The results indicate that consistency has a pronounced effect upon the protective value of mortar and concrete, and that there appears to exist an "optimum consistency" at which the quantity of rust is practically unaffected by time. It was also found that the usual cement contents in reinforced concrete have only a limited effect upon corrosion. It is concluded that water-cement ratio does not in itself control the rate of corrosion of reinforcement.

PROPER SAND GRADING IMPROVES MASS CONCRETE..... 47-9

Price 35 cents.

THOMAS B. KENNEDY—Oct. 1950, pp. 141-152 (V. 47)

Two series of concrete mixtures were designed using 6-in. traprock coarse aggregate and eight separate gradings of natural sand ranging in fineness modulus from 3.60 to 1.35. Tests were made of the plastic concrete, and specimens were cast for tests of compressive strength, resistance to freezing and thawing, and drying shrinkage. Both series of concrete mixtures had a cement content of 2.5 bags per cu yd; one had a normal air content—4½ ± 1½ percent in the portion of the mixture passing the 1½-in. sieve—and the other had a high air content—10 ± 2 percent in the portion of the mixture passing the 1½-in. sieve. Tests indicate that good durability in freezing and thawing can be obtained within the normal air content range with fineness modulus between 2.50 and 2.90. With increased air content, however, the fineness modulus range can be increased to extend from 1.58 to 3.24. Compressive strength was generally affected adversely by increased air content, but not to a serious degree. Drying shrinkage was less with normal air content mixes than with high air content mixtures. It was least when a fineness modulus of 2.52 was used, little difference being apparent between the high and normal air content mixes with this fineness modulus. The air-entraining admixture requirement increased greatly as the fineness modulus of the sand decreased. The water ratio also tended to increase with decreasing fineness modulus.

WATER-SOLUBILITY OF ALKALIES IN PORTLAND CEMENT..... 47-10

Price 35 cents.

J. L. GILLILAND and T. R. BARTLEY—Oct. 1950, pp. 153-160 (V. 47)

In an effort to show correlation of soluble alkalies with alkali-aggregate reaction, the authors hydrated a number of cements for periods up to 90 days and analyzed water extracts of the ground hydrated cement. However, the correlation with expansions of mortar bars prepared with reactive aggregate was not improved by considering water-soluble alkalies rather than total alkalies.

The rate at which the alkalies become water-soluble in hydrating cement indicates that the alkali-bearing phases in cement hydrate quite readily.

NEW PRESTRESSING METHOD UTILIZES VACUUM PROCESS..... 47-11

Price 35 cents.

K. P. BILLNER—Oct. 1950, pp. 161-176 (V. 47)

The method outlined here was developed to simplify prestressing of concrete to make it generally adaptable to American ways of construction. It eliminates costly anchorages; uses large diameter wires (⅝-in. diameter now available on the market), instead of the customary ⅛-in. diameter wire, thus greatly reducing the number of wires required; prestresses all the wires in the building element simultaneously and simplifies forming. A simplified method of design calculations for prestressed concrete and the result of tests of a beam so designed are included.

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DISCUSSION

Discussion closes January 1, 1951

Sept. Jl. '50

Some Observations on the Use of Reinforcing Steel in Concrete Pavements—Bengt F. Friberg

Tests of Paper Molds for Concrete Cylinders—Robert A. Burmeister

Admixtures in Concrete—W. T. Moran, F. H. Jackson, Bruce E. Foster and T. C. Powers

Improved Sonic Apparatus for Determining the Dynamic Modulus of Concrete Specimens—C. E. Goodell

Analysis of Three-Dimensional Beam-and-Girder Framing—Phil M. Ferguson

Discussion closes February 1, 1951

Oct. Jl. '50

Proposed Revision of Specifications for Concrete Pavements and Bases—Committee 617

Linear Traverse Technique for Measurement of Air in Hardened Concrete—L. S. Brown and C. U. Pierson

Influence of the Quality of Mortar and Concrete upon Corrosion of Reinforcement—Rachel Friedland

Proper Sand Grading Improves Mass Concrete—Thomas B. Kennedy

Water-Solubility of Alkalies in Portland Cement—J. L. Gilliland and T. R. Bartley

New Prestressing Method Utilizes Vacuum Process—K. P. Billner

To meet a need for authoritative information on the use of pneumatically-placed mortar, Committee 805 proposes recommendations for shotcreting for adoption as an ACI Standard.

Proposed Recommended Practice for the Application of Mortar by Pneumatic Pressure*

Reported by ACI Committee 805

W. L. CHADWICK
Chairman

J. A. McCRORY R. B. YOUNG

SYNOPSIS

This proposed ACI Standard presents briefly the advantages and disadvantages of pneumatically-placed mortar and establishes recommended practices for placing and mixing shotcrete, qualifications and duties of workmen, preparation of surface before shotcreting, reinforcing, sequence of application, and other items involved in good shotcreting.

INTRODUCTION

Because of its convenience and advantages for various types of work, both for new construction and for repair and maintenance of concrete, brick, masonry and certain steel structures, there has been increased interest during recent years in the use of pneumatically-placed mortar. Mortar so placed, however, has some limitations which should be understood before decision is made to use it. For some work and conditions this material has important advantages; it is convenient, economical and structurally adequate; for other work a part or all of the advantages may be lost. It is important that the best practice be followed in application. Before stating a recommended practice, it is desirable to state the advantages and the limitations of the material to avoid, in so far as possible, misapplication of the material and disappointing results.

Definition

Pneumatically-placed mortar is mortar which is projected by an air jet directly onto the surface to which it is to be applied, irrespective of the type and manufacture of the mixing and placing apparatus. The force of the jet

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This report was approved in form and substance as here submitted by the following members of the committee: W. L. Chadwick, J. A. McCrory and R. B. Young. It is released by the Standards Committee for publication and discussion with view to its consideration for adoption at the 47th annual convention, San Francisco, Calif., Feb 20-22, 1951.

compacts the mortar in place. To avoid the cumbersome term "pneumatically-placed mortar" the word "shotcrete" is used to refer to this material in the remainder of this report.

GENERAL DESCRIPTION OF SHOTCRETING

The processing and placement of shotcrete follows these general steps:

1. Sand and cement are dry-mixed in a standard concrete mixer.
2. The dry mix is fed by gravity into a special mechanical feeder which is hereinafter referred to as a "gun."
3. Compressed air is introduced into the gun and the pressurized mix is fed out of the gun into the delivery hose by a feed wheel.
4. The pressurized dry mix passes through the delivery hose to a special nozzle. In addition to serving as an orifice, this nozzle is provided inside with a special perforated manifold through which water is introduced to the dry mix.
5. The now moistened mix jets immediately onto the object to be shotcreted.

ADVANTAGES OF SHOTCRETE

The advantages of shotcrete are:

1. It requires a relatively small, portable plant for manufacture and placement. This plant consists principally of compressed air and water sources, a standard concrete mixer for dry-mixing the sand and cement, a mechanical feeder (the gun) to feed the dry, mixed material into the delivery hose, hoses for water, air and mixed material, and a nozzle. The material hose conveys the mixed material to the point of application, replacing buckets, buggies, elevators, trucks, pumps, or similar transporting equipment which would otherwise be required. There being no heavy loads to transport, scaffolding requirements are minimum.
2. The method of placement requires a lower water-cement ratio than is normally usable with ordinary concrete placement methods.
3. The density obtained by the method of placement and the stiffness of the mix permit building up of many shapes without the use of forms; where forms are necessary, they may be considerably lighter than those usually required for concrete placement.
4. The dry mix, the method of placement and the necessity for richer mixes result in high compressive strength, and in high density for the given material.
5. Shotcrete is well adapted for use in the construction of the following works:

   New structures—Thin walls, certain roofs, slabs, Imhoff tank baffles, certain reinforced concrete tanks; linings for certain kinds of reservoirs, tunnels, sewers, shafts, canals and pipe; and work of similar extent and section.

   Coatings—Over brick, concrete, steel, masonry. (Under adverse weather conditions on deteriorated concrete surfaces and on many hydraulic

structures the durability will probably be materially less than when used on unexposed surfaces and on nonhydraulic structures.)

Encasement—Of structural steel for fireproofing and for reinforcement; of large steel pipe both inside and out; and of wooden structures for fireproofing such as in mines and some industrial buildings.

Repair—Reservoir linings, surfaces of dams under many conditions, tunnels, shafts, pipe lines, walls, tanks, sewers, canals and most of the work to which shotcrete is adapted for new work. Earthquake damage to masonry structures of all types.

DISADVANTAGES OF SHOTCRETE

The disadvantages of shotcrete are:

1. Shotcrete should be placed by skilled operators only, since both its structural value and the texture of the finished surface are greatly influenced by the skill of the workmen. Structural va'ue and surface are also greatly affected by air pressure, sand grading, sand moisture, water use, humidity and wind.
2. The density of shotcrete is lower than the density of good concrete.
3. While shotcrete is relatively impervious to water, it is more porous than good concrete. (There appears to be evidence that this porosity is favorable to frost resistance of the shotcrete itself.)
4. Shotcrete is more subject to shrinkage cracks than concrete.
5. The natural shotcrete surface is rough and covered with rebound. This condition is objectionable for some classes of work but may be overcome by specially screeding the surface.
6. The placing of shotcrete is relatively dusty, which, with the necessity and expense for cleaning up rebound, is objectionable for some classes of work.
7. Because of the necessity of disposing of rebound clear of the work, shotcrete is applied in relatively thin layers, successive application being necessary for heavy sections.

RECOMMENDED PRACTICE

Materials

Sand—The Engineer should specify whether natural sand or sand prepared by crushing sound stone or gravel shall be used.

Grading—Sand meeting specifications for good concrete is suitable; that is, it should be graded from coarse to fine within the limits shown in the following table:

A.S.T.M. C 33-46	
Pass ⅜ sieve	100 percent
No. 4 sieve	95-100 percent
No. 16 sieve	45- 80 percent
No. 50 sieve	10- 30 percent
No. 100 sieve	2- 10 percent

Fineness modulus not less than 2.50 nor more than 3.30.

Uniformity—For the purpose of controlling the grading of fine aggregate from any one source, a preliminary sample should be submitted prior to actual delivery to the work itself. This sample should be representative of the material it is proposed to furnish. Any shipment of fine aggregate made during the progress of the work which shows a variation of fineness modulus greater than 0.20 either way from a fineness modulus of the preliminary sample should be rejected, unless by changes in proportions it may be readily made to comply with the requirements of this section.

Deleterious substances—Sand should be washed unless specific authority is granted by the Engineer to use unwashed sand. The maximum percentages of deleterious substances should not exceed the following values:

	Percent by weight
Removed by decantation	3
Shale	1
Coal	1
Clay lumps	1

The total of other deleterious substances such as mica, alkali, coated grains and soft flaky particles, should not be more than two percent by weight. The sum of the percentages of shale, coal, clay lumps, soft fragments and other deleterious substances shall not exceed five percent by weight. All sand shall be free from injurious amounts of organic impurities. When subjected to the A.S.T.M. C 40-33 colorimetric test for organic impurities, aggregates producing a color darker than the standard should be rejected.

The sand should contain three to five percent moisture at the time of mixing with cement. If the moisture exceeds five percent, the sand should be dried or allowed to drain until this maximum limit is reached.

Rebound—Rebound is sand and cement which bounces back from the surface where shotcrete is being applied, because of collision with new hard surface, reinforcement or with aggregate particles themselves. The amount varies with cement content, water use, placement angle, shape of surface, size and type of sand used, and air pressure. Because the cement content, cement condition and the fineness modulus in rebound are unpredictable, rebound should not be re-used for shotcrete.

Cement—The cement should be portland cement conforming to the A.S.T.M. specifications for Type II portland cement, Designation C 150-42, and should be subject to the same limitations as to total alkali as first-class concrete.

For volumetric batching, each bag of cement should be considered as 1 cu ft; and, for weight batching, as 94 lb.

Water—The water should be of potable quality.

Reinforcing—Depending on the thickness or extent and nature of the work, reinforcing may be either standard reinforcing steel, or fabric, or expanded metal. Fabric should be welded. Fabric or expanded metal may be self-furring, or may be used with chairs, and may be galvanized or black metal. Specifications or drawings should control each case. It should be emphasized

that the soundest shotcrete will be obtained when reinforcing is selected and designed to cause least interference with placement.

Equipment

Standard mixer—The rotating mixing equipment should be capable of thoroughly dry-mixing the sand and cement within the time available for uniformly and continuously supplying the mixed material to the gun.

Gun—The pneumatic mixing and delivering equipment most extensively used is of the vertical double-chamber type. The upper chamber receives and pressurizes the dry-mix and delivers it to the lower chamber. The lower chamber forces the pressurized mix into the delivery hose by means of a feed wheel. This type of feeder should be of sufficient capacity that the lower chamber may continuously furnish all required material to the delivery hose while the upper chamber receives the recharge.

This equipment must be kept in good condition to avoid reduced pressure and consequent reduced velocity of material delivery. The interior of the drums, feed gearing and valves should be cleaned as often as necessary (at least once every 8-hour shift) to prevent material from caking on critical parts.

Hose and nozzle—Good experience indicates that perhaps the best condition for shotcrete placement is through about 100 ft of material hose, although it is frequently necessary to use longer hose. Placing through as much as 500 ft of hose is practicable if the supply pressures are increased to maintain proper nozzle velocity. For rough or heavy work, a pressure of 50 to 60 lb is required at the nozzle. This pressure may be increased to 75 lb for high lifts or long hose to insure against clogging. Best results for finishing work are to be had when pressures of from 25 to 50 lb are used.

Equipment manufacturers recommend sufficient air pressure at the gun to produce the following material velocities for the nozzle sizes shown:

375 to 500 fps, using $\frac{3}{4}$- or 1-in. nozzles
425 to 550 fps, using $1\frac{1}{4}$-in. nozzle

The nozzles which have given the best performance are of a "pre-mixing" type, with perforated water-feed ring inside the nozzle to direct an even distribution of water through the material at the place of application. These nozzles have a lining which should be replaced when worn to the point where uniformity of material flow is lost.

Air compressor—Any standard type of compressor will be satisfactory if it is of sufficient capacity to provide, without interruption, the pressures and volume of air necessary for the longest hose delivery. In determining compressor capacity, allowance should be made for the air consumed in blowing away rebound, cleaning reinforcing and for incidental uses.

Water pump—Water pressure should be maintained approximately 15 lb higher than the highest air pressure required for placing. Both air and water pressure must be uniformly steady (nonpulsating).

Recommended procedure in placing shotcrete

The work to be done should be fully and clearly described on drawings and in specifications, all of which should be made available to the contractor or work supervisor well before any work is undertaken.

Qualifications and duties of the workmen—Because so much of the quality and satisfactory use of shotcrete depends on the skill of workmen, it is desirable that the foreman, nozzlemen and gunmen, before employment on shotcrete work, give evidence that each has done satisfactory work in similar capacities elsewhere for a sufficient period of time to be fully qualified to properly perform the work in accord with the requirements of the related specifications.

Duties of workmen—The foreman should have had good personal experience, preferably not less than two years as a nozzleman.

The nozzleman should have served a 6 months apprenticeship on similar shotcrete work and should be able to demonstrate by practical test his ability to perform satisfactorily the duties enumerated below.

1. To place the material with a minimum of water at the nozzle, but with enough to insure complete hydration and prevent excessive rebound. Normal rebound will range from 20 to 40 percent of the sand depending on the type of work and on coarseness of sand.
2. To follow a sequence or routine, in placing, which will fill corners with sound material, and cover the reinforcement with the required thickness at each layer.
3. To direct the nozzleman apprentice or helper where to clear away for the subsequent placement.
4. To hold the nozzle at the proper distance (about 3 ft except in close quarters) and at the proper angle to secure maximum compaction with minimum loss of material. This angle should be as nearly normal to the surface as the type of work permits.
5. To direct the crew when to start and stop feeding material, and to stop the work when material is not arriving uniformly at the nozzle.
6. To direct the nozzleman apprentice or helper to cut out sand or slough pockets for replacement after the surrounding shotcrete has taken its initial set. (Immediate replacement is not desirable as a rule, as it may cause additional slough in the surrounding area.)
7. To determine and insure that the air pressure at the gun is uniform and is giving proper velocity to the material at the nozzle, and that the water pressure is sufficiently (15 lb) greater than the air pressure.
8. To bring the shotcrete application to finish lines in a neat workmanlike manner.

The nozzleman apprentice should have had some good experience at similar work. He receives specific orders from the nozzleman but, in general, operates an air nozzle at and around the newly placed work to blow away all loose or porous or shattered material, to prevent its being covered in the progress of the work. Aside from his other specific duties, the nozzleman apprentice or helper helps the nozzleman with the material hose, keeping it advanced for efficient progress.

The gunman operates the mechanical feeder and directs the work of the mixer crew. Utilizing gage readings and his experience, he maintains proper

pressure (45 to 75 lb) on the pneumatic gun to provide proper nozzle velocity. His other most important duty is to assure that the material feed to the nozzle is uniform.

Preparation of the surface for application of shotcrete

Because the final structural value of a shotcrete repair depends upon its bond with the surface on which the mortar is "shot," this surface must be thoroughly cleaned and sound. Surfaces other than concrete surfaces requiring repair should also be clean, and thoroughly compacted (if earth) or well secured (if forms). All coated, scaly, or unsound concrete or masonry should be removed by chipping with pneumatically-operated gads, followed by dressing down to a sound, newly exposed surface utilizing a bush hammer or wire brush, followed by scouring with water and compressed air jet to assure removal of all loose particles. Where exposed structural or reinforcing steel is too corroded to assure a secure bond, the corrosion should be removed by sand-blasting. For this purpose, the shotcreting equipment may be used.

To facilitate bond, the cleaned surface should be moistened not more than an hour prior to placing of shotcrete. This may be accomplished with the nozzle by stopping the flow of material, then scouring the surface with an air and water jet and finally with an air jet alone to remove surplus water.

Drainage

If the surface to be shotcreted is wet from leakage, drain pipes must be placed early enough in advance of the shotcreting to entirely remove the seepage, leaving the surface in approximately the same condition as if moistened as hereinabove noted. (Where structures are subject to freezing, all leakage should be stopped before shotcrete is placed.)

Reinforcing

Reinforcing for new work that is to be shotcreted should be accurately placed in accordance with the drawings, and so well secured in position that no displacement may occur during shotcrete placement.

For repair work, standard reinforcing steel may be welded to other exposed steel within the proposed repair, or may be grouted securely into drilled holes in the common manner of placing anchor dowels. In drilling dowel holes, care should be exercised to avoid damage to existing reinforcing steel. Spacing of steel reinforcing will depend on the location, nature and extent of the repair, and should be shown by drawings. Wire mesh or fabric may be used as suited to the repair. Mesh or fabric should be anchored either by dowels grouted in drilled holes in concrete, or by secure wire ties to other steel. The full area of mesh or fabric should be held firmly in the position shown by the drawings, either by iron wire ties (No. 16 or No. 18), by wire chairs, or by other "furring" methods.

Reinforcing should not be less than $\frac{1}{2}$ in. from the surface against which the shotcrete is to be placed, and there should be not less than $\frac{3}{4}$ in. between the reinforcing and the final surface of the shotcrete.

Reinforcing steel bonds should be lapped 40 diameters. Good practice laps mesh and fabric at least one full mesh. Laps should be tied with No. 16 or No. 18 wire at 12-in. spacing.

Forms and ground wires

Wood forms, where required for support of the full weight of shotcrete, should conform in workmanship and material to supporting forms for concrete, but should be so designed as to permit the escape of placing air and rebound. Wall forms need be placed for only one side of the wall.

Adequate ground (gaging) wires should be used to establish the thickness, surface planes, and finish lines of the shotcrete.

Ground wires should be taut, secure, and true to line and plane.

Proportioning

Except for special work requiring other mixes, the mix should be one part cement to from 4 to 4.5 parts moist sand, volumetrically batch measured. With leaner mixes, the percentage of rebound increases materially.

Mixing

The minimum mixing time for each batch should be stated in the specifications but should be not less than 1½ minutes after the sand and cement are in the drum, when the drum rotates at a peripheral speed of about 200 ft per minute. Each batch should be entirely discharged before recharging is begun.

The mixer should be cleaned at regular intervals (at least once every 8-hour shift) thoroughly enough to remove all adherent material from the mixing vanes and from the drum.

No water should be added to the mix after mixing and before using in the gun. Mixed material that has stood 45 minutes without being used should be rejected. Remixing or tempering should not be permitted.

Sequence of application

Providing rebound is properly cleared away from his work, the nozzleman may work from bottom to top, or vice versa. Corners should be filled first. "Shooting" should be from an angle as nearly normal to the surface as practicable, with the nozzle held approximately 3 ft from the work (except in confined spaces where working room is cramped, where local conditions must control.)

If the flow of material at the nozzle is not uniform, and slugs, sand spots, or wet "sloughs" result, the nozzleman should turn the nozzle away from the work until faulty conditions are corrected. Such defects should be cut out and repaired as the work progresses.

Shotcreting must be temporarily suspended if:

1. High wind occurs which separates the cement from the sand at nozzle;
2. Weather approaches freezing and shotcrete cannot be protected;

3. Rain, other than a very light sprinkle, occurs, otherwise cement is washed out of fresh material.

For best results, shotcrete work should be closely inspected during placement.

Time between coats

The time interval between successive applications in sloping, vertical, or overhanging work must be sufficient to allow initial, but not final, set to develop. At the time the initial set is developing, the surface should be lightly and carefully broomed to remove any laitance and provide a better bond with succeeding applications.

Construction joints

Construction joints, or day's work joints, should be sloped off to a thin, clean, regular edge, preferably at a 45-degree slope. Before placing the adjoining work, the sloped portion and adjacent shotcrete should be thoroughly cleaned as necessary, then wetted and scoured with an air jet.

Surface finish

On completing any surface, the nozzleman should bring the shotcrete to an even plane and to well-formed corners by working up to the ground wires or other thickness or alignment guides, using somewhat lower placing velocity than normal.

If screeding (or rodding) is permitted by the specifications, such work should be done with a thin-edged screed working up against gravity and by a slicing motion to trim off high and expose low spots. A thin finishing or flash coat may be applied to remove rough areas after the ground wires have been removed; or the flash coat may cover the ground wires.

If a float finish is permitted, the final surface should be lightly rubbed with a flat burlap or rubber pad, with circular or spiral motion. No hand patching should be allowed.

If a trowel finish is desired as in water channels, the steel troweling should follow careful screeding in order that wave-free smooth surfaces may be obtained with the least trowel pressure. Troweling should be done not more than one hour after placing.

Integral waterproofing

No admixture should be used unless it is impossible to obtain sufficient fines in the sand, in which case an accepted admixture up to 3 percent of the volume of cement may be used. In cases (such as hazardous tunnel and sewer lining) where a quicker set is essential than can be obtained from the use of high early strength cement, the use of calcium chloride up to 3 percent of the weight of the cement has been used successfully. The calcium chloride, if used, should be added to the mixing water in measured amounts.

Protection from severe weather

Shotcrete should not be placed on a frozen surface nor during freezing weather. As far as possible, "shooting" should be done only in weather when

it is anticipated that the temperature during the following 24 hours will not drop below 32 F.

When shotcreting must be done in freezing temperatures, a sufficiently large area to accommodate the work should be closed in by canvas or similar material, and the enclosure should be heated so that the surface to be shotcreted will be above 32 F when placing is started. The ambient temperature at the work should be maintained above 32 degrees for not less than seven days. Where placing is necessary in freezing weather, the use of high early strength cement is desirable.

Extreme heat, especially when accompanied by hot wind, will cause such quick dryout of a thin shotcrete coating that cracking and separation may be severe. Under such conditions, the shotcrete, immediately upon completion, should be covered with burlap or similar covering, which must be kept continuously moist for 14 days after shotcreting.

Curing under ordinary temperatures

As soon as the fresh shotcreted surface shows the first dry patches, a fine spray of water should be applied to keep it moist. After the surface has hardened, it should be kept moist for seven days.

When the sealing compound method of curing is used, the shotcrete surface should first be sprayed with water; then, directly after, two coats of the sealing compound should be applied. If the shotcrete is exposed to the sun, and the sealing compound is black, the application of sealing coat may be followed by one coat of whitewash.

Tests

Test cylinders should be taken so that they will represent the quality of the shotcrete being placed in the structure by each nozzleman, if more than one nozzleman is used. Each cylinder should be dated, numbered, and the name of the nozzleman noted with the stage of work in the structure.

Test cylinders should be made by shooting shotcrete into a mold of hardware cloth (¾-in. metal mesh) to make cylinders 6 in. in diameter and 12 in. long. The excess material outside the mold is trimmed off with a sharp edged trowel. The open mesh mold is used to permit the escape of air and rebound during placement and provides a cylindrical guide to trim to. About 24 hours after making the cylinders, the hardware cloth form should be removed and the cylinders stored under moist curing conditions at approximately 70 F until tested under A.S.T.M. Designation C 39-42. The tests at the age of 7 days should develop a compressive strength of not less than 3000 psi, and at 28 days not less than 4000 psi.

Measurement

For contract measurement purposes or for unit cost records, all flat work that is continuously of fairly uniform thickness should be measured by the square foot of the minimum required thickness. Irregular shaped work that has little continuity in any major direction should be computed in approximate cubic feet.

It is sometimes desirable, on irregularly shaped work, to use as a basis of payment the number of sacks of cement that have been run through the gun.

Protection of adjacent surfaces and clean-up

During progress of the work, where appearance is important, adjacent walls floors, windows, or any other parts of buildings, ornaments, or grounds which may be permanently discolored, stained, or otherwise damaged by dust and rebound, should be adequately protected and, if contacted, should be cleaned by early scraping, brushing, or washing, as the surroundings permit. Canvas covers should be placed over all equipment which may be injured by cement, dust or water. The same applies to shrubbery.

Inspection

Because of the importance of workmanship affecting the quality of shotcrete, in addition to continuous inspection during placing, all flat surfaces may be sounded with a hammer to detect inadvertent inclusion of rebound or the presence of hollow spots. Any imperfections discovered should be cut out and replaced with sound shotcrete.

Title No. 47-13

A direct, rapid and accurate design method eliminates time consuming "trial and error."

Determining Optimum Cross Sections for Prestressed Concrete Girders*

By FRED J. UZIEL†

SYNOPSIS

General solutions are presented for selecting economically optimum cross sections for prestressed concrete flexural members. Such a solution for simply-supported girders of rectangular cross section, for any span and load, assuming the wires prestressed after setting of the concrete and full dead weight to act during the prestressing operation, leads to a design procedure which, in the case considered, is extremely simple and rapid to apply. It also permits comparisons and studies related to the critical cross section to be made in a more general and conclusive manner. For cross sections other than rectangular, the number of variables making the solution more indeterminate in nature, a similar direct solution is not obtainable. However, a rapid way of obtaining the minimum areas of concrete and steel required is suggested.

INTRODUCTION

The increasing use of prestressing makes the designer of prestressed structures aware of the need for fast and accurate methods of selecting the economical cross section of the prestressed members. Although prestressing has been used to eliminate cracks in tanks, pipes and other structures, its primary advantage lies in the possible economy in materials derived from a reduction in cross section. Flexural prestressed concrete structural members are today economically feasible, provided high quality concrete and steel are at hand, when the size of the job or the duplication of the members to be prestressed is large enough.

In these cases, a fairly accurate knowledge of the economically optimum solution is desirable. Even when the design is based on elastic behavior of the members under working loads and a criterion of allowable stresses, the designer may have a choice of several spans, depths and shapes of members and for each case will have to analyze a number of cross sections before satisfying his stress requirements. Even then he is not sure of getting the best

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design, since a rearrangement of steel and cross-sectional dimensions may result in a more economical member, while stresses are kept the same. The difficulty in rapidly determining the optimum solution lies, of course, not in the interpretation of the basic elastic formulas, but in the increased number of variables and critical conditions.

It is the purpose of this paper to interpret the fundamental formulas in such a way as to make the direct design—in the case of rectangular cross sections—or the knowledge of the minimum quantities of materials required—for the general case—possible, without the need of analyzing trial sections; to show the interdependence of the variables and given quantities and study, in general form, phases of the design of a girder, related to its critical cross section.

In simply-supported girders prestressed after setting of the concrete, critical stress conditions normally occur at top and bottom fibers of the midspan section, (a) immediately after prestressing and (b) on application of full live load with the prestressing force having reached its minimum value. A sufficient load factor at failure must also be obtained and this may be accomplished by a conservative selection of allowable stresses at the full load condition. The position of the cables throughout the length of girder must be made such that the midspan stresses are not exceeded at other sections. Shear stresses and resulting diagonal tension, as well as deflections, must be within safe limits, but will not be found in general to control the selection of cross section. This selection, therefore, is primarily governed by the conditions set above.

On the usual assumptions of elastic behavior and letting gravity loads—*i.e.*, dead weight of girder—act during prestressing, the conditions required are expressed algebraically by:

$$Pe\frac{y_1}{I} - \frac{P}{A} - M_\omega\frac{y_1}{I} \leqq f_t \quad \ldots\ldots (1)$$

$$Pe\frac{y_2}{I} + \frac{P}{A} - M_\omega\frac{y_2}{I} \leqq f_c \quad \ldots\ldots (2)$$

$$-nPe\frac{y_1}{I} + n\frac{P}{A} + (M_\omega + M_W)\frac{y_1}{I} \leqq f'_c \quad \ldots\ldots (3)$$

$$-nPe\frac{y_2}{I} - n\frac{P}{A} + (M_\omega + M_W)\frac{y_2}{I} \leqq f'_t \quad \ldots\ldots (4)$$

Where

P = Total initial prestressing force
A = Area of concrete section
e = Eccentricity of c.g. of cables from c.g. of section
y_1 = Distance of top fiber to c.g. of section
y_2 = Distance of bottom fiber to c.g. of section
I = Moment of inertia of section
M_ω = Maximum moment due to dead load
M_W = Maximum moment due to live load
n = Coefficient of maximum prestress loss
f_c = Allowable compressive stress at prestressing

f_t = Allowable tensile stress at prestressing
f'_c = Allowable compressive stress under full load
f'_t = Allowable tensile stress under full load
Note: f_t, f'_t are positive when tension; f_c, f'_c when compression

RECTANGULAR SECTIONS

If cable holes are neglected

$$\frac{y_1}{I} = \frac{y_2}{I} = \frac{6}{Ah} \quad \ldots\ldots (5)$$

and

$$e = \frac{h}{2} - a \quad \ldots\ldots (6)$$

Where

h = depth of girder
a = distance of c.g. of cables to bottom fiber

Let

W = uniform load per foot-length of girder
ω = specific weight of concrete
L = span of girder

The first of the original conditions may be written

$$P\left(\frac{h}{2} - a\right)\frac{6}{Ah} - \frac{P}{A} - \frac{A\omega L^2}{8} \times \frac{6}{Ah} \leqq f_t \quad \ldots\ldots (7)$$

or

$$\frac{P}{A}\left(2 - \frac{6a}{h}\right) - \frac{3\omega L^2}{4h} \leqq f_t \quad \ldots\ldots (8)$$

Similarly, the other conditions may be written

$$\frac{P}{A}\left(4 - \frac{6a}{h}\right) - \frac{3\omega L^2}{4h} \leqq f_c \quad \ldots\ldots (9)$$

$$-n\frac{P}{A}\left(2 - \frac{6a}{h}\right) + \frac{3\omega L^2}{4h} + \frac{3WL^2}{4Ah} \leqq f'_c \quad \ldots\ldots (10)$$

$$-n\frac{P}{A}\left(4 - \frac{6a}{h}\right) + \frac{3\omega L^2}{4h} + \frac{3WL^2}{4Ah} \leqq f'_t \quad \ldots\ldots (11)$$

These conditions assume that the c.g. of cables is below the lower third point of the section or

$$a \leqq \frac{h}{3} \quad \ldots\ldots (12)$$

The unknown quantities in the conditions above are A,P,a and h. It may be shown that the four conditions cannot be equated simultaneously. For instance, if Eq. (8), (9) and (11) are equated, by subtracting member to member (8) from (9):

$$2\frac{P}{A} = f_c - f_t$$

and Eq. (10), by subtracting members of Eq. (11), becomes equivalent to

$$2n\frac{P}{A} \leqq f'_c - f'_t$$

and therefore to

$$n(f_c - f_t) \leqq f'_c - f'_t \quad \ldots\ldots (13)$$

which depends entirely on the given quantites and is not necessarily an equation.

A maximum of three of the four original conditions may then be simultaneously equated. Since the depth of girder is often controlled by considerations other than structural economy, such as architectural or clearance requirements, h will be considered known or assumed in any particular case.

Within a range of the given quantities, which will be defined, equating conditions (8), (9) and (11) will always give simultaneously minimum values for P and A, for any given value of h. Assume that smaller values of f_t, f_c and f'_t were needed to equate these conditions. Solving for A and P, the following values are obtained:

$$A = \frac{3WL^2}{4(nf_c + f'_t)h - 3\omega L^2 (1 - n)} \quad \ldots\ldots (14)$$

$$P = \frac{3WL^2 (f_c - f_t)}{8 (nf_c + f'_t)h - 6\omega L^2 (1 - n)} \quad \ldots\ldots (15)$$

By inspection it is seen that A varies inversely with f_c and f'_t and P inversely with f_t and f'_t. P also varies inversely with f_c when

$$\frac{dP}{df_c} \leqq 0$$

which may be written

$$(1 - n)\, 3\omega L^2 - 4h\,(nf_t + f'_t) \geqq 0 \quad \ldots\ldots (16)$$

When this condition is satisfied, the maximum values of f_c, f_t and f'_t must be used to give minimum A and P values, therefore the allowable stresses must be used in equating conditions (8), (9) and (12).

The design procedure suggested is therefore to solve for the values of A, P and the position of the cables, a, by selecting a depth of girder, h, and using Eq. (8), (9) and (11). Letting for convenience

$$k = \frac{3\omega L^2}{4f_c h} \quad ; \quad q = \frac{f_t}{f_c} \quad ; \quad q' = \frac{f'_t}{f_c} \quad \ldots\ldots (17)$$

the values A, P and a are

$$A = \frac{W}{\omega} \times \frac{1}{\dfrac{n + q'}{k} - (1 - n)} \quad \ldots\ldots (18)$$

$$P = \frac{f_c}{2} (1 - q)\, A \quad \ldots\ldots (19)$$

$$a = \frac{h}{3(1 - q)} \times (1 - 2q - k) \quad \ldots\ldots (20)$$

This solution is subject to conditions of Eq. (12), (13) and (16) which may now be written

$$k + q' \geqq 0 \quad \ldots\ldots (12a)$$

$$f'_c = n(f_c - f_t) + f'_t \quad \ldots\ldots (13a)$$

$$(1 - n)k - (nq + q') \geqq 0 \quad \ldots\ldots (16a)$$

Considering the above conditions for their practical significance they will be found to apply to the great majority of applications. In application, con-

dition (12a) need not be checked provided the c.g. of cables is below the lower third point of the section determined. Condition (13a) may be used to find the actual stress under full load and checking it against the allowable stress, as illustrated in the example following. Condition (16a) is sometimes not satisfied and an alternate solution is then required if the minimum value of P is to be obtained. However, the possible reduction in steel is small, while the concrete section has to be increased manyfold from its minimum value as obtained from Eq. (18), therefore, the alternate solution will be found of little practical importance and will be omitted in this presentation.

The majority of problems can be solved by Eq. (18) to (20). These can for convenience be put in the form of simple graphs or nomographs, the number of variables having been reduced to a minimum.

It will be noted from consideration of these equations that the maximum h will result in smaller A and P in an almost linear relation. This should be one of the considerations in the selection of the depth of girder. The linearity of the relation permits the approximate determination of A and P for any depth of girder after the equations have been once applied for any one value of h.

The position of the cables as defined by Eq. (20) is not always practical. The value of a may sometimes be negative or too small to place the cables. The minimum acceptable value of a must then be used for an optimum solution. The value of A does not have to be changed since equating simultaneously Eq. (9) and (11) eliminates both P and a. The value of P has to be increased to a value:

$$P' = P \times \frac{2h - 3a}{2h - 3a'} \qquad (21)$$

where P and a are the values computed from Eq. (19) and (20) and a' is the corrected value of a. The increase of P required is ordinarily small.

Example

To determine the optimum rectangular cross section of a simply-supported prestressed concrete girder of 80-ft span with a uniform load of 1 ton per ft, the following stresses being allowed: $f_c = 2000$ psi (comp.); $f_t = 150$ psi (ten.); $f'_c = 1800$ psi (comp.); $f'_t \mp -100$ psi (comp.) Also given: Allowable wire stress = 150,000 psi; 15 percent maximum loss of prestress, or $n = 0.85$; depth to span ratio, 1/16, or $h = 5$ ft; wires of 3/16-in. diameter to be used, $a_s = 0.0276$ sq in.

The actual stress f'_c is found from Eq. (13) and checked:

$$f'_c = 0.85\,(2000 - 200) - 100 = 1430 < 1800$$

Compute constants from Eq. (17):

$$k = \frac{3 \times 150 \times 80^2}{4 \times 5 \times 2000 \times 144} = 0.50 \quad ; \quad q = \frac{150}{2000} = 0.075 \quad ; \quad q' = \frac{-100}{2000} = -0.05$$

From Eq. 18:

$$A = \frac{2000}{150} \times \frac{1}{\dfrac{0.85 - 0.05}{0.50} - 0.15} = 9.18 \text{ sq ft}$$

From which the width b of section required is found:

$$b = \frac{9.18}{5} \times 12 = 22.04 \text{ in.} \quad ; \quad \text{say } b = 22 \text{ in.}$$

From Eq. (19):

$$P = \frac{2000}{2}(1 - 0.075) \times 22 \times 60 = 1{,}220{,}000 \text{ lb}$$

From which the number of wires required may be found:

$$\text{Number}_1 = \frac{1{,}220{,}000}{150{,}000 \times 0.0276} = 294 \text{ wires}$$

To locate the cables from Eq. (20):

$$a = \frac{5 \times 12}{3(1 - 0.075)} \times (1 - 0.15 - 0.50) = 7.56 \text{ in.; say } a = 7\tfrac{1}{2} \text{ in.}$$

For the sake of illustration, assume that 7.5 in. is not enough to place the 294 wires and that 9.5 in. is the minimum distance from bottom fiber at which the c.g. of an anticipated number slightly greater than 294 may be placed. The same concrete section must be used, but the value of P and therefore the number of wires must be changed according to Eq. (21):

$$\text{Number}_2 = 294 \times \frac{2 \times 60 - 3 \times 7.5}{2 \times 60 - 3 \times 9.5} = 313 \text{ wires}$$

Design formulas Eq. (18) to (20) neglect the openings for cables, an error being introduced in the stresses which in many cases is negligible, or may become appreciable in others. The increments of stresses due to cable holes may be estimated after one design and subtracted on the second design from the stresses f_c, f_t, f'_t to be used. These increments may, however, be known in advance, provided the value of

$$c = \frac{\text{Area of cable holes}}{A} \qquad (22)$$

is known. The value of c is approximately constant for a particular type of cable and is about 2.5 percent. By considering the displacement of the c.g. of section due to the reduction of area by cA, the increments which should be added algebraically before these are used in Eq. (18) to (20) may be derived and are found to be:

$$\Delta f_t = -\frac{2}{3} ck\,(2 + k) f_c \qquad (23)$$

$$\Delta f_c = -\frac{4}{3} c\,(1 + k + k^2) f_c \qquad (24)$$

$$\Delta f'_i = +\frac{2}{3} nc\,(1 - k^2) f_c \qquad (25)$$

The value of k may be computed from Eq. (17) on the basis of the given allowable f_c, or if more accuracy is required, on the basis of the corrected f_c on second evaluation. In the case where the cable holes are grouted after prestressing, only corrections for f_c and f_t by Eq. (23) and (24) are required.

The application of these corrections may be illustrated in the example previously solved, assuming that $c = 2.5$ percent is representative of the assembly of wires used.

$$f_t = -\frac{2}{3} \times 0.025 \times 0.50\,(2 + 0.50) \times 2000 = -42 \text{ psi}$$

$$f_c = -\frac{4}{3} \times 0.025\,(1 + 0.50 + 0.25) \times 2000 = -117 \text{ psi}$$

$$f'_t = +\frac{2}{3} \times 0.85 \times 0.025\,(1 - 0.25) \times 2000 = +32 \text{ psi}$$

The design stresses must then be:

$f_c = 2000 - 117 = 1883$ psi
$f_t = 150 - 42 = 108$ psi
$f'_t = -100 + 32 = -78$ psi

The solution based on these stresses is:

$k = 0.53$; $q = 0.0574$; $q' = -0.0414$; $A = 9.70$ sq ft; b = 23¼ in;
$P = 1{,}240{,}000$ lb; number of wires = 300; $a = 7\frac{1}{2}$ in.

Other types of loading

So far, girder design under uniform load has been considered. However, Eq. (18) to (20) may be applied to any other type of loading if W is taken as the equivalent uniform load, provided the maximum live load moment occurs at or near the center of the span. For girders of varying depth, the derived equations have to be altered by changing the value M_ω. Similarly, for cases where dead load is not assumed acting during prestressing, $M_\omega = 0$ must be used in the first two original conditions given in Eq. (1) and (2).

Slabs

The design of one-way prestressed slabs can be made readily by solving Eq. (18) where $A = bh = h$ per foot-width of slab for h, using W as the live load per sq ft of slab. The optimum depth of slab required is then given by:

$$4(nf_c + f'_t)\,h^2 - (1 - n)\,3\omega L^2 h - 3WL^2 = 0 \quad \ldots\ldots (26)$$

where length units are in feet. For an approximation:

$$h = \frac{L}{2}\sqrt{\frac{3W}{nf_c + f'_t}} \quad \ldots\ldots (26a)$$

The value of P is then found from Eq. (19), the location of cables by Eq. (20).

Example

To design a slab of 40-ft span, with a 150 psf live load and the same allowable stresses as in the previous example. The optimum depth is found from:

$$4 \times 144\,(0.85 \times 2000 - 100)h^2 - (1 - 0.85) \times 3 \times 150 \times 40^2 h - 3 \times 150 \times 40^2 = 0$$

$$1.28\,h^2 - 0.15\,h - 1 = 0$$

$h = 0.945$ ft say $h = 11\frac{3}{8}$ in.

$$P = \frac{2000}{2}\,(1 - 0.075) \times 12 \times 11.375 = 126{,}000 \text{ lb per ft}$$

$$\text{Number of wires} = \frac{126{,}000}{150{,}000 \times 0.0276} = 30 \text{ wires per ft}; \quad a = 3\frac{1}{4} \text{ in.}$$

LOCATION OF CABLES

Contrary to conventional girders, a change of cross section along the length of girder is done of necessity, rather than for structural economy only, to keep

fiber stresses along the girder within the allowable limits. An important incidental advantage is the reduction of shear stresses which may be obtained by proper placement of the cables.

There are three ways of providing for the decrease of moments toward the supports: (1) bending up the cables, (2) decreasing the girder depth toward the ends, and (3) decreasing length of cables and bending up at intermediate points. These methods have been used separately or in combination with each other, each having different advantages. The last method has the advantage of possible reduction in steel and of better control of shear stresses. However, the complication of detailing, the difficulty of anchoring cables at the top face without destroying appearance and usefulness and the loss of prestressing due to friction of cables are its principal disadvantages. Varying the girder depth has the advantage that straight cables may be used, friction losses eliminated and placing of steel is easier. However, formwork is made expensive, the method being better adaptable to precast members. The more common procedure is the bending up of cables, which will be considered in more detail.

When cables are made parabolic for a girder of uniform cross section subject to uniform live loads, all fiber stresses along the girder are safe. This is so because these stresses may be expressed, under dead load, as:

$$\pm \frac{P}{A} + \frac{y}{I}(Pe - M_\omega) \quad \ldots\ldots (27)$$

and under full load as:

$$\pm \frac{nP}{A} + \frac{y}{I}(M_\omega + M_W - nPe) \quad \ldots\ldots (28)$$

The values of n,P,A,y and I are constants throughout the length of girder, while the values under parenthesis are composed of members varying parabolically from a maximum at mid-span to a minimum at supports. The magnitude of the top and bottom fiber stresses will therefore vary parabolically from a maximum at center to a minimum at supports. It is not, however, required that the bending-up of cables be in parabolic form. If e', M'_ω, M'_W are the eccentricity of cables, the dead load and live load moment at any section, respectively, it is required and sufficient that:

$$Pe' - M'_\omega \leqq Pe - M_\omega \quad \ldots\ldots (29)$$

$$M'_\omega + M'_W - nPe' \leqq M_\omega + M_W - nPe \quad \ldots\ldots (30)$$

or that

$$M_\omega - M'_\omega \leqq P(e - e') \leqq \frac{M_\omega - M'_\omega + M_W - M'_W}{n} \quad \ldots\ldots (31)$$

The maximum value of n must be used for a critical condition:

$$M_\omega - M'_\omega \leqq P(e - e') \leqq (M_\omega + M_W) - (M'_\omega + M'_W) \quad \ldots\ldots (32)$$

This condition can best be represented graphically, by drawing to a convenient scale the $\frac{M_\omega}{P}$ and $\frac{M_\omega + M_W}{P}$ diagrams, so that their apexes coincide.

When moving live load is used, the maximum $\frac{M_\omega + M_W}{P}$ must be used. A

reference line may be drawn to the same scale at a distance e from the common apex. The condition above is then represented by the hatched area, within which the c.g. of the cables must be kept (Fig. 1).

For girders of rectangular cross section selected by optimum design, condition (32) can also be expressed algebraically. For cases where $f_t = f'_t = 0$, by substitution of Eq. (17) to (20):

$$k\left(1 - 2\frac{x}{L}\right)^2 \leqq \frac{3(e - e')}{h} \leqq (1 + k)\left(1 - 2\frac{x}{L}\right)^2 \quad \ldots\ldots (33)$$

which allows the immediate computation of maximum and minimum positions of the c.g. of cables at any section x distance from the supports.

SHEAR STRESSES AND DIAGONAL TENSION

The case of a simply-supported girder of rectangular uniform cross section with the c.g. of cables running parabolically from an eccentricity e at midspan to 0 at supports will be considered.

The shear force due to prestressing at supports is of opposite sign to shear forces due to loads and its minimum value is $\frac{nPe}{L/4}$. The total shear at supports, under full uniform load is

$$V = \frac{W + \omega A}{2} L - \frac{4nPe}{L} \quad \ldots\ldots (34)$$

The maximum unit shear at center of the section at supports is

$$v = \frac{3V}{2A} = \frac{3}{4}\left(\frac{W}{A} + \omega\right)L - \frac{6nPe}{AL} \quad \ldots\ldots (35)$$

The superimposed compression is

$$c = n\frac{P}{A} \quad \ldots\ldots (36)$$

Assuming $f_t = f'_t = 0$ and optimum design

$$\frac{P}{A} = \frac{f_c}{2} \quad ; \quad \frac{W}{A} = \omega\left(\frac{n}{k} - 1 + n\right) \quad ; \quad e = \frac{h}{6}(1 + 2k)$$

Substituting in Eq. (35) and (36)

$$v = \frac{nf_c h}{2L} \quad ; \quad c = n\frac{f_c}{2}$$

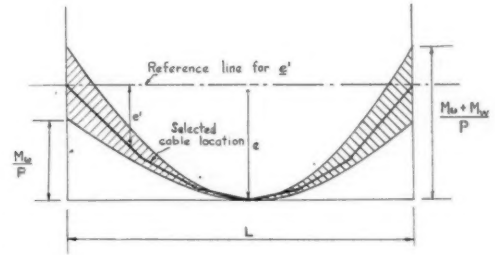

Fig. 1—Diagram for locating cables

The principal tension is

$$p_t = -\sqrt{v^2 + \frac{c^2}{4}} + \frac{c}{2} = -\frac{nf_c}{4}\left[\sqrt{\frac{4h^2}{L^2} + 1} - 1\right] \quad \ldots\ldots (37)$$

Similarly, at prestressing:

$$v = \frac{3}{4}\omega L - \frac{6Pe}{AL} = -\frac{f_c h}{2L}$$

$$c = \frac{f_c}{2}$$

$$p_t = -\frac{f_c}{4}\left[\sqrt{\frac{4h^2}{L^2} + 1} - 1\right] \quad \ldots\ldots (38)$$

which is the critical principal tension.

Since the allowable diagonal tension is normally specified as a percentage of compressive stress, it is seen that the depth-span ratio is the only factor affecting the magnitude of the tensile principal stress.

$$\text{For } \frac{h}{L} = \frac{1}{10} \quad : \quad p_t = -0.005 f_c$$

$$\text{For } \frac{h}{L} = \frac{1}{20} \quad : \quad p_t = -0.0012 f_c$$

These stresses are safe for any concrete since the allowable diagonal tension may be about 2 percent of the crushing strength, or about 5 percent of f_c, while the computed stresses are smaller. Principal stresses at other points of the section at supports are smaller since the unit shear is smaller while c is the same. They are also smaller at center points of other sections along the length of girder since c is the same while v varies lineally from its value at supports to zero at center. If other allowable stresses f'_t are used or the cables deviate from the exact parabolic position the shear stresses are somewhat different but remain in the same range.

It may be concluded that for a girder of rectangular cross section, selected by optimum design and with steel near a parabolic position, shear stresses are safely carried by the concrete and no shear reinforcement is required.

DEFLECTIONS

Assumptions similar to those of the previous section are made. The eccentricity of steel at any section x distance from the supports is

$$e' = \frac{4e}{L^2} \times (L - x)\, x \quad \ldots\ldots (39)$$

Similarly the bending moment under full load is

$$M_x = \frac{4x\,(L - x)}{L^2} \times (M_\omega + M_W - nPe) \quad \ldots\ldots (40)$$

For optimum design:

$$M_\omega + M_W - nPe = \frac{A\omega + W}{8} L^2 - \frac{nf_c}{2} \times A \times \frac{h}{6}(1 + 2k)$$

$$= \frac{Anf_c h}{12} = \frac{W}{12\omega} \times \frac{f_c h}{\frac{1}{k} - \frac{1}{n} + 1} \quad \ldots\ldots (41)$$

The influence of n is small in the above expression and $n = 1$ may be taken as an approximation, from which

$$M_\omega + M_W - nPe = \frac{W}{12\omega} \times \frac{3\omega L^2}{4f_c h} \times f_c h = \frac{WL^2}{16} \quad \ldots\ldots (42)$$

The maximum deflection is therefore

$$\partial_{max} = -\frac{1}{EI} \int \int_o^{L/2} \frac{W}{4} x(L - x) dx \quad \ldots\ldots (43)$$

Solving the double integral, the constants of integration being found from equating to zero the slope at mid-span and deflection at supports:

$$\partial_{max} = -\frac{5WL^4}{768EI} \quad \ldots\ldots (44)$$

The deflection due to live load may be found similarly and is twice the value above. Under dead load and prestressing the girder has therefore an upward deflection equal to the final downward deflection under full load expressed by Eq. (44).

COMPARISON OF PRESTRESSED AND CONVENTIONAL GIRDERS

Respective quantities of materials for prestressed and conventional girders, for any given span and intensity of applied load, will be compared on the basis of rectangular cross sections of same depth, assuming that the same compressive stress and no tension are allowed. The effect of plastic flow and shrinkage are neglected for the conventional section design, which will be based on ACI procedure.

$$\text{Modular ratio} = m = \frac{30{,}000}{\text{Ult. comp. str.}} = \frac{30{,}000}{\frac{f_c}{0.45}} = \frac{13{,}500}{f_c}$$

or

$$mf_c = 13{,}500$$

The position of the neutral axis is given by:

$$k_1 = \frac{1}{\frac{f_s}{mf_c} + 1} = \frac{13{,}500}{f_s + 13{,}500}$$

The sum of horizontal forces gives:

$$A_s f_s = \frac{1}{2} f_c k_1 db \quad \ldots\ldots (45)$$

where d, the effective depth of reinforcement, may be written:

$$d = rh = r\frac{A}{b} \quad \ldots\ldots (46)$$

The sum of moments about the neutral axis gives:

$$A_s f_s \left(d - \frac{k_1 d}{3} \right) = \frac{WL^2}{8} + \frac{A\omega L^2}{8} \quad \ldots\ldots (47)$$

Substituting the value of $A_s f_s$ from above and solving for A:

$$A = \frac{W}{\omega} \times \frac{1}{\frac{40{,}500\ (f_s + 9000)}{(f_s + 13{,}500)^2} \times \frac{r^2}{k} - 1}$$

where $k = \frac{3\omega L^2}{4 f_c h}$. For $f_s = 20{,}000$ psi:

$$A = \frac{W}{\omega} \times \frac{1}{1.048 \frac{r^2}{k} - 1}$$

The value of A_s is then found from substitution:

$$A_s = \frac{W}{\omega} \times \frac{f_c}{40{,}000} \times \frac{1}{\frac{2.59r}{k} - \frac{2.48}{r}} \quad \ldots\ldots (48)$$

These expressions are similar to the ones derived for prestressed members and for equal L, W, f_c and h may be compared directly:

$$R_c = \frac{\text{Concrete area, conventional}}{\text{Concrete area, prestressed}} = \frac{\frac{n}{k} - 1 + n}{\frac{1.048r^2}{k} - 1} \quad \ldots\ldots (49)$$

$$R_s = \frac{\text{Steel area, conventional}}{\text{Steel area, prestressed}} = \frac{f_s \text{ (prestr.)}}{20{,}000} \times \frac{\frac{n}{k} - 1 + n}{\frac{2.59r}{k} - \frac{2.48}{r}} \quad \ldots\ldots (50)$$

The value of r is made as great as possible and is about 0.90 to 0.95. For these values and $n = 0.85$, the ratios (49) and (50) are respectively:

$$R_c = \frac{0.85 - 0.15k}{0.85 - k} \quad \text{and} \quad \frac{0.85 - 0.15k}{0.945 - k}$$

$$R_s : \frac{f_s}{20{,}000} = \frac{0.85 - 0.15k}{2.33 - 2.76k} \quad \text{and} \quad \frac{0.85 - 0.15k}{2.46 - 2.61k}$$

These ratios are plotted in Fig. 2 for values of the variable k. With increasing values of W and L, more shear reinforcement is needed in the conventional design and the actual value of R_s is increased.

The important conclusion to be drawn from the diagram is that above k values of about 0.5, the comparison favors the prestressed girder. For values of k smaller than 0.3 the saving of materials may be not enough to overbalance the additional construction costs of prestressing.

GIRDERS OF ANY SHAPE OF CROSS SECTION

The selection of optimum cross section for I-, T-, box- or channel-shaped girders cannot be done readily by the use of direct formulas, the number of unknown variables making the problem indeterminate. It is, however, possible to arrive at the values of the minimum areas of concrete and steel required, without the uncertainties and the tedious work involved in a series of analysis of trial cross sections.

The original four conditions [Eq. (1) to (4)] contain six unknowns, P, A, e, y_1, y_2 and L. Three equations may be written, since one of the conditions—Eq. (3) for ordinary values of the variables—depends only upon the stresses assumed. The solution is indeterminate, but by assuming the depth of girder and the ratio of P/A it is possible to solve for minimum P and A.

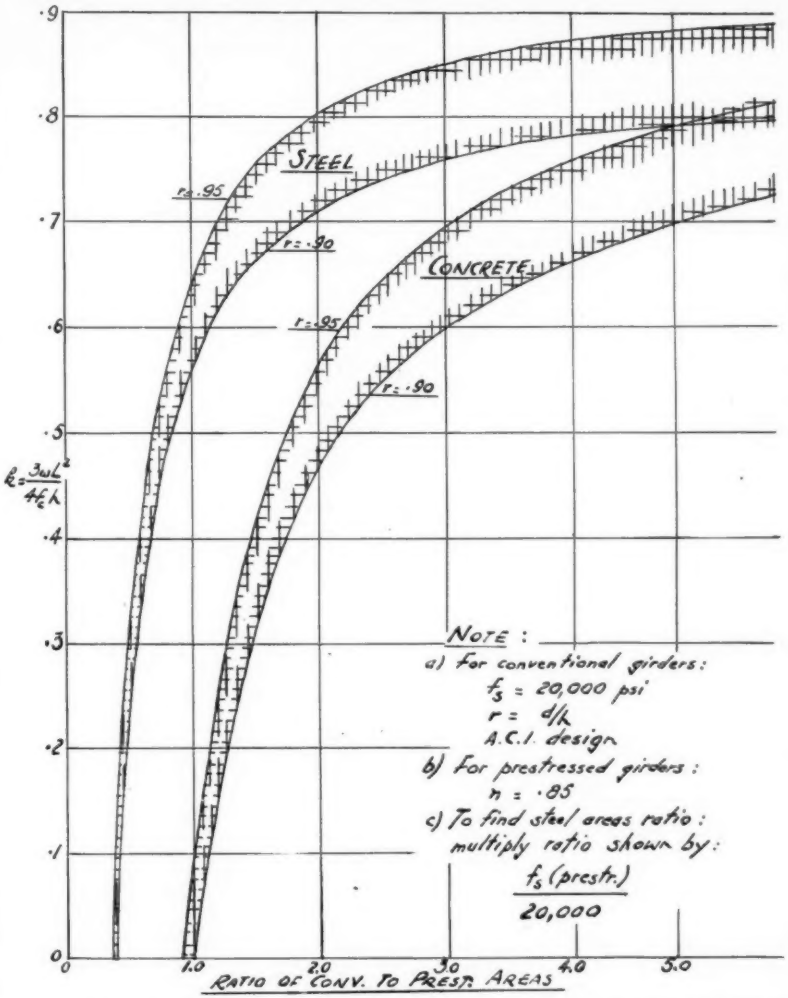

Fig. 2—Comparison between conventional and prestressed concrete rectangular sections for simply-supported girders

Multiplying Eq. (1) by y_2 and Eq. (2) by y_1 and subtracting member to member:

$$\frac{P}{A}(y_1 + y_2) = y_1 f_c - y_2 f_t$$

and since $y_1 + y_2 = h$

solving for y_2:

$$y_2 = \frac{f_c - \frac{P}{A}}{f_c + f_t} \times h \quad \ldots\ldots (51)$$

Similarly by solving Eq. (2) and (4), eliminating the unknown quantity $\frac{y_2}{I}$ and solving for A, letting $M_\omega = \frac{A\omega L^2}{8}$:

$$A = \frac{f_c - \frac{P}{A}}{\frac{Pe}{A} - \frac{\omega L^2}{8}} \times \frac{(1-n)M_\omega + M_W}{nf_c + f'_t} \quad \ldots\ldots (52)$$

It will be noted that the unknown value of M_ω in the right-hand member is multiplied by $(1 - n)$ which is about 0.15. Therefore, an estimated value of M_ω can safely be used for Eq. (52). It is also noted that for any given P/A the higher value of e will result in minimum A, therefore minimum P. The following procedure is therefore suggested for evaluating the minimum P and A required:

The maximum h consistent with clearance or other considerations is selected. Reasonable values of P/A are then tried and y_2 computed from Eq. (51). The values of P/A should be of the order of $f_c/2$. The maximum probable eccentricity is then estimated and values of A computed from Eq. (52), from which corresponding P values are found. By inspection of the groups of P and A values the optimum combination of P and A is found.

The method outlined may be used in combination with the design of a girder of rectangular cross section to find whether or not it is advantageous to go to a more irregular shape.

If the design of an irregular shape is attempted with minimum A and P, as found above, the required moment of inertia is found from Eq. (2) by substitution of y_2:

$$I = \frac{Pe - M_\omega}{f_c + f_t} \times h \quad \ldots\ldots (53)$$

The quantities A, I, h and the position of the center of gravity are the quantities solved and an elementary shape with four fundamental dimensions may be determined. For practical reasons, such as minimum web and flange dimensions, the optimum section thus found is not always practicable.

The method outlined will be illustrated by application to the 80-ft span girder previously designed for a rectangular cross section.

Example

A wide range of P/A ratios is used for illustration, actually a smaller range but a larger number of ratios may be found appropriate. Computations are given in Table 1.

The curves of the values of A and P are shown in Fig. 3 from which it may be seen that minimum A = 960 sq in. and minimum P = 970,000 lb, in-

TABLE 1—DESIGN DATA FOR IRREGULARLY SHAPED GIRDERS

	P/A					
	400	600	800	1000	1200	1400
$f_c - \frac{P}{A}$, lb	1600	1400	1200	1000	800	600
y_2, in.*	44.6	39.1	33.5	27.9	22.3	16.7
e, in.†	39.6	34.1	28.5	22.9	17.3	11.7
$\frac{P}{A}e$, lb	15,820	20,420	22,770	22,900	20,750	16,410
$\frac{P}{A}e - \frac{\omega L^2}{8}$, lb	5820	10,420	12,770	12,900	10,750	6410
$\frac{f_c - P/A}{Pe/A - \frac{\omega L^2}{8}}$, in.	0.275	0.134	0.094	0.0775	0.0744	0.0935
A, sq in.§	3560	1735	1216	1003	963	1210
P, kips	1425	1040	974	1003	1160	1695

* y_2 from Eq. (51).

† e by subtracting from y_2 the minimum estimated end distance of the c.g. of cables to the lower fiber. Taken as 5 in.

‡ $\frac{\omega L^2}{8} = \frac{150 \times 80^2}{8 \times 12} = 10{,}000$ lb.

§ From Eq. (52) where:

$$Mw = 2000 \times 80^2 \times \frac{12}{8} = 19{,}200{,}000 \text{ in.-lb}$$

$$(1 - n)\, M\omega = \text{approx. } 0.15 \times \frac{1000}{144} \times 150 \times 80^2 \times \frac{12}{8} = 1{,}500{,}000 \text{ in.-lb}$$

Assuming 1000 sq in. area of concrete—smaller than the rectangular area required of 1320 sq in.

$$\frac{(1 - n)\, M\omega + Mw}{nf_c + f'_t} = \frac{1{,}500{,}000 + 19{,}200{,}000}{0.85 \times 2000 - 100} = 12{,}950 \text{ cu in.}$$

stead of 1320 sq. in. and 1,220,000 lb, respectively, obtained with a rectangular cross section. The two minimum values for A and P are not, however, simultaneous and the designer must select the optimum P/A ratio from practical considerations. For instance, it may be practical to use 260 wires, which corresponds to a prestressing force of 1,080,000 lb. From the curves it is found that $P/A = 1120$ psi and $A = 965$ sq in. The required moment of inertia of the section is then found from Eq. (53):

$$I = \frac{1{,}080{,}000 \times 19.5 - \frac{965}{144} \times 150 \times \frac{80^2}{8} \times 12}{2000 - 150} \times 60 = 370{,}000 \text{ in.}^4$$

CONCLUSIONS

For simply-supported prestressed girders of rectangular cross section, a general solution was expressed from which it is possible to find the least areas of steel and concrete and the position of steel for any given span and load, provided the allowable stresses are known and the percentage of prestress loss assumed. One set of equations was found to be adequate for the majority of applications. The solution thus expressed is found to be adequate for direct, rapid and accurate design, eliminating the need of "trial and error" and permitting conclusive statements on shear stresses, etc., and comparisons of prestressed to alternate girders to be made in general form.

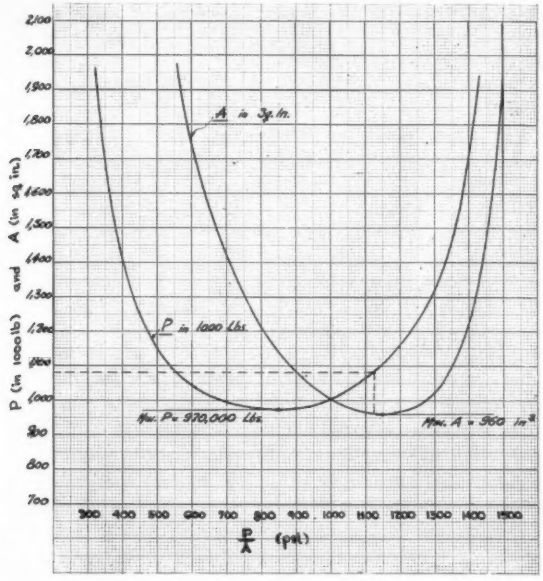

Fig. 3—Curves for selecting optimum quantities of steel and concrete for girders of any shape of cross section

For the general case of prestressed girders of any shape of cross section, a procedure of establishing the optimum quantities was suggested: (a) To select the optimum combination of P and A, together with the required I and position of c.g., from which a cross section may be derived. (b) To check the accuracy of cross sections designed by other methods which are approximate. (c) To justify whether or not a cross section of irregular shape is appropriate.

Consideration of the equations expressed for the case of rectangular cross section, permits the study of the effect of the variables and known quantities upon the economy of the girder. This has not been discussed in detail but an indication of these variations may be given. Both steel and concrete areas are found to vary approximately with the square of the span, with the intensity of live load and inversely to the depth of girder. The unit weight ω of concrete will be found to have only a slight effect upon the cross section required, while the effect of n is great and increases with larger values of k and smaller values of q'. For example, for $k = 1$ and $q' = 0$, the percentage saving in steel and concrete when n is increased from 0.85 to 0.90 is about 15 percent, emphasizing that accurate knowledge of n is desirable. The area of concrete is approximately inversely proportional to the allowable compression f_c, which, however, does not affect substantially the amount of steel required. The effect of allowable tension is substantial in many cases, *i.e.*, for $k = 0.5$, a change of f'_t from 0 to 0.15 f_c reduces steel and concrete required by 15 percent and for $k = 0.8$, by 17 percent.

Cheaper winter concreting made possible through use of steam curing

Steam Curing Protects Winter Concreting*

By C. O. CRANE†

SYNOPSIS

The use of live steam for protecting newly placed concrete from freezing weather and for providing initial curing has resulted in excellent concrete in the Enders Dam spillway at no greater cost than less desirable dry heating methods. This brief paper describes in detail the methods used.

Fig. 1—General view of the spillway stilling pool and left spillway channel wall showing the location of the 150 hp oil field-type boiler used to generate steam. The boiler was enclosed in a temporary shack for the purpose of conserving heat and improving the working conditions of the firemen, particularly those men working on the night shifts. Steam pipes leading from the boiler were boxed in straw for insulation.

INTRODUCTION

Enders Dam, on Frenchman Creek in the southwest corner of Nebraska, is being built by the Bureau of Reclamation as one unit of the Missouri River Basin Project. Much of the 48,000 cu yd of concrete required for the outlet works and spillway (Fig. 1) of this rolled earth-fill dam was placed during freezing weather.

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The use of salamanders on the Enders work during the winters of 1948 and 1949 was not fully satisfactory. A considerable number of expensive tarpaulins were accidentally burned, together with some forms, leaving the concrete temporarily exposed. It was found difficult to maintain a uniform temperature over the entire concrete surface when salamanders were used. Often areas of the concrete near the salamander were hot (and dry) while other areas some distance from the salamander approached the minimum permissible temperature. Dry heat introduced a troublesome curing problem and was therefore not as desirable as the moist heat produced by live steam.

The Bureau specifications require that concrete placed in freezing weather shall not be permitted to fall below 50 F nor rise above 90 F for the first three days, and that protection from freezing shall continue for the next 11 days. During the month of December, 1948, heating of the aggregates was unnecessary to secure placing temperatures of 50 F or more. However, heating of the mixing water was required and two oil burning hot water heaters, having a combined capacity capable of raising the temperature of 3300 gal. of water 50 degrees per hour, were used. One pair of water heating units was located on each of the two 34-cu ft mixers.

Curing is inevitably involved in winter protection procedures, and either 14 days of water curing or the use of an approved white-pigmented sealing compound* is required. Results with the latter are always improved by an initial period of water curing. Live steam applied during the initial period controlled both temperature and curing in a manner conducive to the development of excellent concrete.

PROCEDURE

Immediately after the concrete was placed and finished, the new concrete was protected by live steam as a means of temperature control as well as of curing. No effort was made to effect early maturity of the concrete by excessive heat from steam. Only sufficient steam was released to maintain the concrete temperature above 50 F. Generally, this temperature ranged from 50 to 65 F. These relatively low temperatures avoided the necessity of cooling the concrete slowly after the steam was discontinued and minimized the danger of shrinkage cracks in the concrete resulting from subsequent temperature drops; furthermore, such temperatures were more favorable to the ultimate strength of the concrete than higher temperatures.† The steam also provided a uniform temperature over the entire concrete surface which is difficult to obtain with individual hot air heaters. Housing for the steam was made from old forms or a timber framework covered by tarpaulins and therefore inexpensive (Fig. 2). Drops in atmospheric temperature were readily compensated for by increasing the amount of steam generated.

*Blanks, R. F., Meissner, H. S., and Tuthill, L. H., "Curing Concrete with Sealing Compounds," ACI JOURNAL, Apr. 1946, *Proc.* V. 42, p. 493.

†Tests at the Bureau of Reclamation Concrete Laboratory, Denver, Colo., have indicated that the ultimate compressive strength of concrete cured at lower temperatures is higher than that steam cured at high temperatures for similar short early curing periods.

Where the steam pipes passed over a newly finished concrete surface, a piece of burlap was laid over the concrete to prevent condensation from dripping onto the concrete and marring the finish. Usually the steam was discontinued on most floor slabs during the warm part of the day following placement, after which the concrete was painted with white-pigmented sealing compound for curing. After the sealing compound had dried, the entire block was covered with about one ft of straw (Fig. 3). Records show a concrete temperature of 50 F or more was maintained at all times where this amount of straw was used. It is believed that the use of live steam for temperature control for the first 20 hours or more was good procedure, afford-

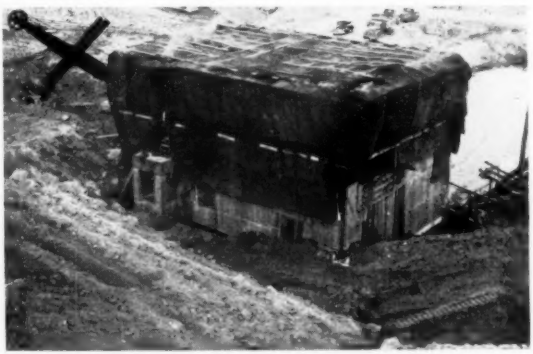

Fig. 2—The concrete for the top of the valve house superstructure, inclusive of the roof slab, was placed in one operation. This portion of the structure was completely surrounded on the outside by a single live steam line with several openings along each of the four walls. After this preparation, the entire concrete placement was securely wrapped in tarpaulins.

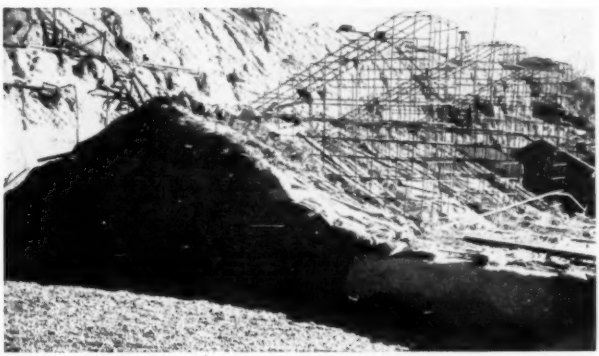

Fig. 3—Floor slab sections with a portion of the downstream dentates in the stilling pool were steam cured for approximately 20 hours immediately following concrete placement. After discontinuance of the application of steam, the surfaces were painted with a white-pigmented sealing compound and covered with straw. The straw was held in place by tarpaulins or burlap, weighted or tied down as shown in the photograph. Construction joints such as those shown in the photograph, where the steel is in place for the remaining portion of the dentate, were cured by wet sand buried under the straw.

ing excellent temperature and moisture control during the most critical period in the life of the concrete. Covering the concrete with straw subsequent to the early steam curing provided adequate protection against freezing at a minimum cost (Fig. 4).

Fig. 4—View of the spillway stilling pool, looking upstream, showing completed upstream dentates and a portion of the downstream dentates under construction. The newly placed concrete of the lower dentates is being protected by a covering of straw and burlap.

On wall sections where large forms were involved the steam was applied at the outside of the form. The concrete was thus well protected and removal of the form was left to the contractor's discretion. Wall sections were generally steam cured for 72 hours or more and after removal of the forms, the concrete was painted with the white-pigmented sealing compound for further curing, and subsequently heated with salamanders to prevent freezing.

CONCLUSION

The contractor feels that on this job the use of steam was less expensive than the use of stoves or salamanders—all things considered. Moreover, indications to date are that with the steam protection and curing a concrete was obtained that had greater freedom from shrinkage cracking than that placed in warm weather under "favorable" conditions. The concrete placed and cured under the conditions outlined above when tested for durability withstood over 1200 alternate cycles of freezing and thawing and had an average compressive strength of more than 4000 psi.

Title No. 47-15

Continuous beams, with axes curved in plan, must be designed for both internal bending and twisting moments in addition to transverse shear.

Analysis of Continuous Circular Curved Beams*

By BECLA VELUTINI†

SYNOPSIS

Continuous circular curved beams can be analyzed easily by the moment distribution method if both bending and torsional end couples are considered. Formulas and tables are presented for circular curved beams of constant cross section that give the relations between the end moments and end torques and the corresponding rotations of the end sections. A proposed method of procedure is illustrated in which the bending end couples are kept separate from the torsional couples. The mathematical operations are not difficult as the convergence is rapid, but attention must be paid to the sign convention which must be definite and easy to apply.

INTRODUCTION

Continuous beams whose axes are curved in plan are sometimes used to advantage in buildings and special structures. Such members will ordinarily be continuous and monolithic with the columns as, for example, spandrel beams of reinforced concrete buildings with curved exterior walls.

The analysis of beams curved in plan differs from the ordinary analysis of continuous beams because of the torsional moments that are induced by vertical loads even though these loads pass through the axis of the member. Therefore, it is necessary to design such members for both internal bending and twisting moments as well as transverse shear.

Formulas and tables are presented for circular curved beams of constant cross section that give the relations between the end moments and end torques and the corresponding rotations of the end sections. These relations have been derived from the expression for the total strain energy due to bending and torsion by means of Castigliano's theorem. From these strain relations between end forces and end rotations the familiar stiffness and carry-over factors of the moment distribution method are derived. In this problem, however, there are two stiffness coefficients (for moment and torque) and six carry-over factors which give the simultaneous change in all end moments and torques

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for any end rotation. The use of the two stiffness coefficients in obtaining distribution factors and of the carry-over factors in determining the corresponding changes in the other end moments and torques is illustrated by a numerical example.

NOTATIONS AND ASSUMPTIONS

The following notations are used:

M = moment about radial axis. Its subscript indicates its location. The sign is positive if clockwise.
T = torque. Its subscript indicates its location. The sign is positive if clockwise.
F = vertical force. Its subscript indicates its location.
r = radius of curvature of the member.
β = angle spanned by the member.
α = angular distance of any section of the member, measured counterclockwise from the support.
E = modulus of elasticity in tension or compression.
I = moment of inertia of the cross section of the member.
G = modulus of rigidity.
GJ = torsional stiffness.
k = EI/GJ.
ω = angle of rotation due to bending. Its subscript indicates its location, its sign is positive when clockwise.
ϕ = angle of rotation due to torque. Its subscript indicates its location, its sign is positive when clockwise.
Δ = vertical displacement. Its subscript indicates its location.
q = vertical uniform load per unit length of beam.
C_m = fixed-end bending moment coefficient.
C_t = fixed-end torque coefficient.

The following assumptions are used in the solution:

(a) Hooke's law is valid.
(b) The principle of superposition is valid.
(c) The deformations due to the vertical shear forces are negligible.
(d) The angle of bending per unit length of beam varies as M/EI.
(e) The angle of twist per unit length of beam varies as T/GJ.

The value of k, which depends on the shape of the cross section, can be calculated for rectangular reinforced concrete beams in the following manner:*

$$k = \frac{EI}{GJ} = \frac{(2.25)\ E\ b\ d^3/12}{E\ b^3\ d^3}\ (3.58)\ (b^2 + d^2)$$

$$k = 0.671\left(1 + \frac{d^2}{b^2}\right)$$

b is the width, and d the depth of the cross section.

$\frac{E}{G}$ was assumed equal to 2.25.

DEVELOPMENT OF EQUATIONS

A beam curved in plan with end b free and end a held against any type of movement is shown in Fig. 1.

*See "Design of Reinforced Concrete in Torsion," by Paul Andersen, *Trans.*, A.S.C.E., V. 103.

At b (Fig. 1) a moment M_b, a torque T_b, and a vertical force F_b are acting on the beam as indicated by the vectors.

At any section an angular distance α from b the bending moment M_α and the twisting moment T_α are equal to

$$M_\alpha = M_b \cos \alpha + T_b \sin \alpha - F_b r \sin \alpha$$

$$T_\alpha = -M_b \sin \alpha + T_b \cos \alpha + F_b r (1 - \cos \alpha)$$

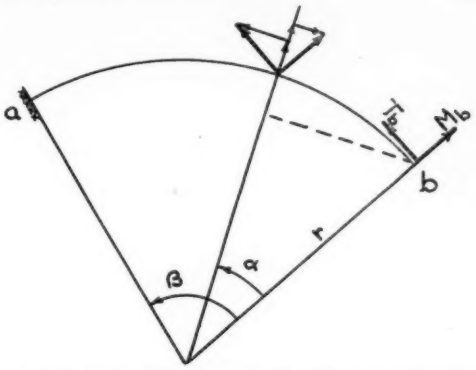

Fig. 1—Beam curved in plan with one fixed and one free end

The strain energy due to the forces acting on the beam, neglecting the effect of vertical shear, will be:

$$U = \int_0^\beta \frac{M_\alpha^2 r \, d\alpha}{2EI} + \int_0^\beta \frac{T_\alpha^2 r \, d\alpha}{2GJ}$$

The rotation ω_b at b in the direction of M_b is, by Castigliano's theorem,

$$\omega_b = \frac{\partial U}{\partial M_b}$$

Hence

$$\omega_b = \int_0^\beta \left[M_b \cos \alpha + T_b \sin \alpha - F_b r \sin \alpha \right] \frac{(\cos \alpha) r \, d\alpha}{EI} +$$

$$\int_0^\beta \left[-M_b \sin \alpha + T_b \cos \alpha + F_b r (1 - \cos \alpha) \right] \frac{(-\sin \alpha) r \, d\alpha}{GJ}$$

Calling $\frac{EI}{GJ} = k$,

$$\omega_b = \frac{r}{2EI} M_b \left[\beta (k + 1) - \sin \beta \cos \beta (k - 1) \right] - \frac{r}{2EI} T_b \sin^2 \beta (k - 1) +$$

$$\frac{r^2}{2EI} F_b \left[\sin^2 \beta (k - 1) + 2k (\cos \beta - 1) \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The rotation ϕ_b at b in the direction of T_b is

$$\phi_b = \frac{\partial U}{\partial T_b}$$

or:

$$\phi_b = \int_0^\beta (M_b \cos \alpha + T_b \sin \alpha - F_b r \sin \alpha) \frac{r \sin \alpha \, d\alpha}{EI} +$$

$$\int_0^\beta \left[-M_b \sin \alpha + T_b \cos \alpha + F_b r (1 - \cos \alpha) \right] \frac{r \cos \alpha \, d\alpha}{GJ}$$

$$\phi_b = -\frac{r}{2EI} M_b \sin^2\beta\,(k-1) + \frac{r}{2EI} T_b \left[\beta\,(k+1) + \sin\beta\cos\beta\,(k-1)\right] + \frac{r^2 F_b}{2EI}\left[2k\sin\beta - \beta\,(k+1) - \sin\beta\cos\beta\,(k-1)\right] \quad \ldots\ldots(2)$$

The vertical displacement Δ_b at b in the direction of F_b will be

$$\Delta_b = \frac{\partial U}{\partial F_b}$$

$$\Delta_b = \int_o^\beta -(M_b\cos\alpha + T_b\sin\alpha - F_b\,r\sin\alpha)\frac{r^2\sin\alpha\,d\alpha}{EI} + \left[-M_b\sin\alpha + T_b\cos\alpha + F_b\,r\,(1-\cos\alpha)\right]\frac{r^2\,(1-\cos\alpha)\,d\alpha}{GJ}$$

$$\Delta_b = \frac{r^2}{2EI} M_b\left[\sin^2\beta\,(k-1) + 2k\,(\cos\beta - 1)\right] + \frac{r^2 T_b}{2EI}\left[2k\sin\beta - \beta\,(k+1) + \sin\beta\cos\beta\,(k-1)\right] + \frac{r^2 F_b}{2EI}\left[\beta\,(k+1) + \sin\beta\cos\beta\,(k-1) - 4k\sin\beta + 2k\,\beta\right] \quad \ldots\ldots(3)$$

From these three equations the values of M_b, T_b, F_b can be obtained for any known values of the displacement and rotations at b. It will be convenient to make the following substitutions in the above equations:

$A = \beta\,(k+1) - \sin\beta\cos\beta\,(k-1)$
$B = \sin^2\beta\,(k-1)$
$C = \sin^2\beta\,(k-1) - 2k\,(1-\cos\beta)$
$D = \beta\,(k+1) + \sin\beta\cos\beta\,(k-1)$
$L = 2k\sin\beta - \beta\,(k+1) - \sin\beta\cos\beta\,(k-1)$
$H = \beta\,(k+1) + \sin\beta\cos\beta\,(k-1) - 4k\sin\beta + 2k\,\beta$

Eq. (1), (2) and (3) can now be expressed in the form:

$$M_b A - T_b B + F_b\,r\,C = \omega_b\frac{2EI}{r} \quad \ldots\ldots(4)$$

$$-M_b B + T_b D + F_b r\,L = \phi_b\frac{2EI}{r} \quad \ldots\ldots(5)$$

$$M_b C + T_b L + F_b r\,H = \Delta_b\frac{2EI}{r^2} \quad \ldots\ldots(6)$$

If the end a is fixed and end b is held against twist and vertical displacement, ($\phi_b = 0$; $\Delta_b = 0$) then for any rotation $\omega_b = 1$, the value of M_b will be the coefficient of stiffness, S_m, for bending in the member. If M_b is expressed in terms of $\frac{EI}{r}$, the stiffness factor S_m is obtained as follows.

For $\phi_b = 0$; $\Delta_b = 0$ Eq. (5) and (6) give:

$$M_b B = T_b D + F_b r\,L \quad \ldots\ldots(7)$$

$$-M_b C = T_b L + F_b r\,H \quad \ldots\ldots(8)$$

From Eq. (7) and (8):

$$T_b = \left(\frac{B\,H + L\,C}{D\,H - L^2}\right) M_b \quad \ldots\ldots(9)$$

$$F_b r = -\frac{D\,C + B\,L}{D\,H - L^2} M_b \quad \ldots\ldots (10)$$

$$M_b A - \left(\frac{B\,H + L\,C}{D\,H - L^2}\right) M_b B - \left(\frac{D\,C + B\,L}{D\,H - L^2}\right) M_b C = \omega_b \frac{2EI}{r}$$

$$M_b = \frac{2\,(D\,H - L^2)}{A\,(D\,H - L^2) - B\,(B\,H + L\,C) - C\,(D\,C + B\,L)} \omega_b \frac{EI}{r} \quad \ldots\ldots (11)$$

If now $\Delta_b = 0$, $\omega_b = 0$, and $\phi_b = 1$, the twisting moment T_b can be expressed in terms of $\frac{EI}{r}$ as follows:

$$T_b B = M_b A + F_b r\, C \quad \ldots\ldots (12)$$

$$-T_b L = M_b C + F_b r\, H \quad \ldots\ldots (13)$$

From Eq. (12) and (13):

$$M_b = \frac{B\,H + L\,C}{A\,H - C^2} T_b \quad \ldots\ldots (14)$$

$$F_b r = \frac{-(A\,L + B\,C)}{A\,H - C^2} T_b \quad \ldots\ldots (15)$$

Introducing these values in Eq. (2) gives:

$$T_b = \frac{2\,(A\,H - C^2)}{A\,(D\,H - L^2) - B\,(B\,H + L\,C) - C\,(D\,C + B\,L)} \phi_b \frac{EI}{r} \quad \ldots\ldots (16)$$

The carry-overs to the end a of the member can now be calculated from the expressions:

$$M_a = M_b \cos\alpha + T_b \sin\alpha - F_b r \sin\alpha \quad \ldots\ldots (17)$$

$$T_a = -M_b \sin\alpha + T_b \cos\alpha + F_b r\,(1 - \cos\alpha) \quad \ldots\ldots (18)$$

by making $\alpha = \beta$.

The carry-over factors between the two ends can be obtained from Eq. (17) and (18) by substituting the values of T_b and F_b [Eq. (9) and (10)] in terms of M_b; or M_b and F_b [Eq. (14) and (15)] in terms of T_b as given above. When these substitutions are made, all stiffness and carry-over factors are known. These values can be calculated directly from the following equations:

Stiffness factor S_m for an applied end moment =

$$\frac{2\,(D\,H - L^2)}{A\,(D\,H - L^2) - B\,(B\,H + L\,C) - C\,(D\,C + B\,L)} \quad \ldots\ldots (19)$$

Stiffness factor S_t for an applied torque =

$$\frac{2\,(A\,H - C^2)}{A\,(D\,H - L^2) - B\,(B\,H + L\,C) - C\,(D\,C + B\,L)} \quad \ldots\ldots (20)$$

Ratio of torque to applied end moment at end 1 (from Eq 9) is:

$$m_1 t_1 = \frac{B\,H + L\,C}{D\,H - L^2} \quad \ldots\ldots (21)$$

Ratio of end moment to applied torque at end 1 (from Eq. 14) is:

$$t_1 m_1 = \frac{B\,H + L\,C}{A\,H - C^2} \quad \ldots\ldots (22)$$

Ratio of the end moment at end 2 to an applied end moment at end 1 (from Eq. 17) is:

$$m_{12} = \cos\beta + \left(\frac{B\,H + L\,C}{D\,H - L^2}\right)\sin\beta + \left(\frac{D\,C + B\,L}{D\,H - L^2}\right)\sin\beta \quad \text{.....(23)}$$

Ratio of torque at end 2 to applied end moment at end 1 (from Eq. 18) is:

$$m_1\,t_2 = -\sin\beta + \left(\frac{B\,H + L\,C}{DH - L^2}\right)\cos\beta - \left(\frac{DC + BL}{DH - L^2}\right)(1 - \cos\beta) \quad \text{.....(24)}$$

Ratio of end moment at end 2 to applied torque at end 1 (from Eq. 17) is:

$$t_1\,m_2 = \sin\beta + \left(\frac{BH + LC}{AH - C^2}\right)\cos\beta + \left(\frac{AL + BC}{AH - C^2}\right)\sin\beta \quad \text{.....(25)}$$

Ratio of torque at end 2 to applied torque at end 1 (from Eq. 18) is:

$$t_{12} = \cos\beta - \left(\frac{BH + LC}{AH - C^2}\right)\sin\beta - \left(\frac{AL + BC}{AH - C^2}\right)(1 - \cos\beta) \quad \text{.....(26)}$$

Numerical values of the various coefficients for values of k equal to 1, 2, 3, 4 and 10 have been computed from Eq. (19)-(26) inclusive. These values are tabulated in Table 1. For general use, diagrams should be prepared from these values so that any interpolation can be made more accurately.

TABLE 1—STIFFNESS AND CARRY-OVER COEFFICIENTS

	β	10°	30°	60°	90°	120°	150°	180°
$k = 1$	S_m	22.86085	7.41104	3.42554	1.99862	1.25228	0.80802	0.53523
	S_t	5.79035	2.07669	1.27918	1.08037	0.99707	0.93085	0.85354
	m_{t1}	0.04366	0.13088	0.26134	0.38898	0.50368	0.59847	0.63662
	m_{12}	−0.50226	−0.52031	−0.58184	−0.68148	−0.80936	−0.93667	−1.00000
	m_{t2}	0.00012	0.00234	0.01991	0.07046	0.17648	0.36211	0.63662
	t_{m1}	0.17238	0.46677	0.69978	0.71960	0.63637	0.51950	0.39921
	t_{12}	0.98494	0.87724	0.62682	0.41074	0.28170	0.23428	0.25414
	t_{m2}	−0.00021	−0.00861	−0.05342	−0.13034	−0.22160	−0.31432	−0.39921
$k = 2$	S_m	22.83125	7.36013	3.32045	1.88369	1.14889	0.72190	0.46170
	S_t	2.92279	1.12266	0.78133	0.70820	0.66626	0.61785	0.55848
	m_{t1}	0.04362	0.13033	0.25713	0.37716	0.48639	0.57766	0.63662
	m_{12}	−0.50317	−0.52659	−0.60297	−0.71400	−0.83933	−0.94954	−1.00000
	m_{t2}	0.00015	0.00347	0.02789	0.09116	0.20800	0.38931	0.63662
	t_{m1}	0.34072	0.85443	1.09272	1.00319	0.88873	0.67495	0.52330
	t_{12}	0.97028	0.77717	0.43755	0.23929	0.16879	0.14903	0.24006
	t_{m2}	−0.00105	−0.02286	−0.11855	−0.24248	−0.35883	−0.44994	−0.52630
$k = 3$	S_m	22.81008	7.30222	3.22500	1.78679	1.06515	0.65348	0.40593
	S_t	1.96742	0.80228	0.60448	0.56164	0.52599	0.48162	0.42978
	m_{t1}	0.04357	0.12977	0.25345	0.36820	0.47325	0.56604	0.63662
	m_{12}	−0.50396	−0.53299	−0.62137	−0.73867	−0.85873	−0.95671	−1.00000
	m_{t2}	0.00017	0.00462	0.03483	0.10687	0.22857	0.40448	0.63662
	t_{m1}	0.50515	1.18116	1.35219	1.17137	0.95834	0.76804	0.60130
	t_{12}	0.95601	0.69469	0.32664	0.16862	0.14176	0.18189	0.25936
	t_{m2}	−0.00236	−0.04176	−0.18590	−0.33999	−0.46284	−0.54917	−0.60130
$k = 4$	S_m	22.78493	7.23264	3.13791	1.70278	0.99464	0.59733	0.36219
	S_t	1.48989	0.64000	0.50951	0.47692	0.44261	0.40050	0.35369
	m_{t1}	0.04357	0.12925	0.25021	0.36117	0.46405	0.55862	0.63662
	m_{12}	−0.50478	−0.53877	−0.63758	−0.75802	−0.87232	−0.96130	−1.00000
	m_{t2}	0.00024	0.00560	0.04095	0.11919	0.24291	0.41418	0.63662
	t_{m1}	0.66625	1.46064	1.54099	1.28950	1.04281	0.83294	0.65191
	t_{12}	0.94202	0.62551	0.25507	0.13606	0.13920	0.19620	0.28002
	t_{m2}	−0.00354	−0.06306	−0.25230	−0.42556	−0.54582	−0.61757	−0.65191
$k = 10$	S_m	22.66817	6.87742	2.73088	1.34919	0.71937	0.39532	0.21996
	S_t	0.63004	0.33880	0.30333	0.27616	0.24402	0.21089	0.17885
	m_{t1}	0.04343	0.12647	0.23673	0.33830	0.43933	0.54113	0.63662
	m_{12}	−0.50952	−0.56939	−0.70507	−0.81295	−0.90882	−0.97211	−1.00000
	m_{t2}	0.00052	0.01108	0.06645	0.15125	0.28139	0.43703	0.63662
	t_{m1}	1.56254	2.56726	2.13124	1.65277	1.29512	1.01436	0.78294
	t_{12}	0.86488	0.37214	0.09528	0.12522	0.19354	0.27170	0.35282
	t_{m2}	−0.01816	−0.22412	−0.56425	−0.77799	−0.82951	−0.81922	−0.78294

FIXED-END MOMENTS DUE TO UNIFORM LOADS

The end moments for a beam curved in plan when both ends are fixed and loaded uniformly with q per unit length of beam will now be determined.

The vertical reaction F_b at end b (Fig. 2) is equal to

$$F_b = \frac{q\,r\,\beta}{2}$$

and the distance OG from the center of curvature to the center of gravity of arc bm is

$$OG = \frac{r \sin \alpha/2}{\alpha/2}$$

At a distance $r\alpha$ from b;

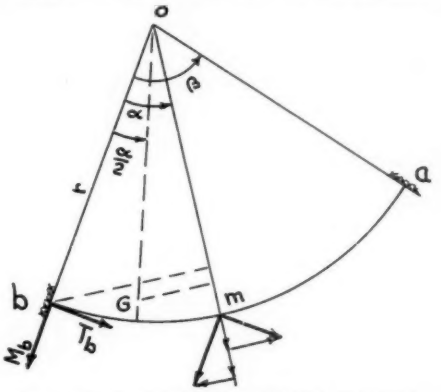

Fig. 2—Beam curved in plan with both ends fixed

$$M_a = M_b \cos\alpha + T_b \sin\alpha + F_b\, r \sin\alpha - q\, r\, \alpha \frac{r \sin \alpha/2}{\alpha/2} \times \sin \alpha/2$$

$$M_a = M_b \cos\alpha + T_b \sin\alpha + \frac{q\,r^2}{2}\beta \sin\alpha - q\,r^2\,(1 - \cos\alpha)$$

$$T_a = T_b \cos\alpha - M_b \sin\alpha - F_b\,(r - r\cos\alpha) + q\,r\,\alpha\,(r - \frac{r \sin \alpha/2}{\alpha/2} \cos \alpha/2)$$

$$T_a = T_b \cos\alpha - M_b \sin\alpha - \frac{q\,r^2}{2}\beta\,(1 - \cos\alpha) + q\,r^2\,(\alpha - \sin\alpha)$$

Using an auxiliary moment $M'_b = 1$ at b, at a distance α; $M'_a = \cos\alpha$; $T'_a = -\sin\alpha$. As the beam is fixed:

$$\int_o^\beta M'_a\, M_a \frac{r\,d\alpha}{EI} + \int_o^\beta T'_a\, T_a \frac{r\,d\alpha}{GJ} = \omega_b = 0$$

or

$$\int_o^\beta \frac{r}{EI}\left[M_b \cos\alpha + T_b \sin\alpha + \frac{q\,r^2}{2}\beta \sin\alpha - q\,r^2\,(1 - \cos\alpha)\right]\cos\alpha\,d\alpha -$$

$$\int_o^\beta \frac{r}{GJ}\left[T_b \cos\alpha - M_b \sin\alpha - \frac{qr^2}{2}\beta\,(1 - \cos\alpha) + qr^2\,(\alpha - \sin\alpha)\right]\sin\alpha\,d\alpha = 0$$

which gives

$$\tfrac{1}{2} M_b\,[\beta\,(k+1) - \sin\beta\cos\beta\,(k-1)] - \tfrac{1}{2}\,T_b \sin^2\beta\,(k-1) +$$
$$\tfrac{1}{2} qr^2\,[\beta\,(k+1)\,\beta\,k\,(1+\cos\beta) - \tfrac{1}{2}\,\beta \sin^2\beta\,(k-1) -$$
$$2\sin\beta\,(k+1) - \sin\beta\cos\beta\,(k-1)] = 0$$

If an auxiliary torque $T''_b = 1$ is applied, the moments at a distance α from b will be: $M''_a = \sin\alpha$; $T''_a = \cos\alpha$. As the beam is fixed,

$$\int_o^\beta M''_a\, M_a \frac{r\,d\alpha}{EI} + \int_o^\beta T''_a\, T_a \frac{r\,d\alpha}{GJ} = \phi_b = 0$$

$$\int_0^\beta \frac{r}{EI}\left[M_b \cos\alpha + T_b \sin\alpha + \tfrac{1}{2} qr^2 \beta \sin\alpha - qr^2 (1 - \cos\alpha)\right] \sin\alpha \, d\alpha +$$

$$\int_0^\beta \frac{r}{GJ}\left[T_b \cos\alpha - M_b \sin\alpha - \tfrac{1}{2} qr^2 \beta (1 - \cos\alpha) + qr^2 (\alpha - \sin\alpha)\right] \cos\alpha \, d\alpha = 0$$

which gives

$$-\tfrac{1}{2} M_b \sin^2\beta (k-1) + \tfrac{1}{2} T_b \left[\beta (k+1) + \sin\beta \cos\beta (k-1)\right] +$$
$$\tfrac{1}{2} qr^2 \left[\tfrac{1}{2} \beta^2 (k+1) + \tfrac{1}{2} \beta \sin\beta \cos\beta (k-1) - \sin^2\beta (k-1) + k\beta \sin\beta - 2(k-1)(1 - \cos\beta)\right] = 0$$

Canceling out the $\frac{1}{2}$, and substituting as follows:

$$A' = \beta (k+1)$$
$$B' = \sin^2\beta (k-1)$$
$$C' = \sin\beta \cos\beta (k-1)$$

$$D' = \beta (k+1) + k\beta (1 + \cos\beta) - \frac{\beta}{2} \sin^2\beta (k-1) - 2 \sin\beta (k+1) - \sin\beta \cos\beta (k-1)$$

$$E' = \tfrac{1}{2} \beta^2 (k+1) + \frac{\beta}{2} \sin\beta \cos\beta (k-1) - \sin^2\beta (k-1) + k\beta \sin\beta - 2(k+1)(1 - \cos\beta)$$

The equations will now be:

$$M_b (A' - C') - T_b B' + qr^2 D' = 0$$
$$-M_b B' + T_b (A' + C') + qr^2 E' = 0$$

Solving for M_b and T_b gives

$$M_b = qr^2 \left[\frac{B' E' + D' (A' + C')}{-A'^2 + B'^2 + C'^2}\right] \text{ and } T_b = qr^2 \left[\frac{B' D' + E' (A' - C')}{-A'^2 + B'^2 + C'^2}\right]$$

After substituting the values of the various terms, the fixed end bending moments and torques become:

$$M_b = M_{Fb} = qr^2 \left[\frac{2(k+1)\sin\beta - k\beta(1 + \cos\beta)}{\beta(k+1) - \sin\beta (k-1)} - 1\right] = qr^2 C_m \ldots\ldots\ldots (27)$$

$$T_b = T_{Fb} = qr^2 \left[\frac{2(k+1)(1 - \cos\beta) - k\beta \sin\beta}{\beta(k+1) - \sin\beta (k-1)} - \frac{\beta}{2}\right] = qr^2 C_t \ldots\ldots\ldots (28)$$

Numerical values of C_m and C_t are tabulated in Table 2 for various values of k. Diagrams should be prepared from these values before any interpolation is made.

APPLICATION TO THE MOMENT DISTRIBUTION METHOD

The use of the moment distribution method for the analysis of a continuous beam curved in plan will now be illustrated. As there are two sets of distribution and carry-over factors, one for bending and one for torque, it is necessary to have a sign convention that can be applied to the end of any span (see Fig. 3).

The end moments are positive when, looking outwards from the center of curvature, they produce a clockwise rotation of the section.

The torques are positive when, looking along the tangent to the beam in a counterclockwise sense, they cause a clockwise rotation of the section.

TABLE 2—FIXED-END MOMENT COEFFICIENTS FOR UNIFORM LOADS

	β	30°	60°	90°	120°	150°	180°
$k = 1$	C_m	−0.02316	−0.09601	−0.22676	−0.42300	−0.68501	−1.00000
	C_t	−0.00007	−0.00169	−0.01216	−0.04782	−0.13346	−0.29756
$k = 2$	C_m	−0.02325	−0.09711	−0.23004	−0.42742	−0.68744	−1.00000
	C_t	−0.00011	−0.00233	−0.01544	−0.05547	−0.14253	−0.29756
$k = 3$	C_m	−0.02335	−0.10135	−0.23244	−0.43097	−0.68879	−1.00000
	C_t	−0.00014	−0.00478	−0.01784	−0.06162	−0.14755	−0.29756
$k = 4$	C_m	−0.02343	−0.09887	−0.23428	−0.43211	−0.68964	−1.00000
	C_t	−0.00017	−0.00336	−0.01968	−0.06360	−0.15073	−0.29756
$k = 10$	C_m	−0.02390	−0.10209	−0.23999	−0.43712	−0.69163	−1.00000
	C_t	−0.00034	−0.00523	−0.02539	−0.07227	−0.15817	−0.29756

The vertical forces are positive when directed upwards.

By using this convention, it was found that the carry-over factors should be used with the signs shown in Fig. 4.

NUMERICAL EXAMPLE

The continuous beam shown in Fig. 5 is fixed at points 1 and 4 and is supported by square columns at points 2 and 3. The end spans are loaded with a uniform load of one kip per ft and the center span with two kips per ft. Besides the dimensions given in Fig. 5, the following values will be used:

Length of columns = 14 ft 0 in.
Moment of inertia of beams = I_b = unity
Moment of inertial of columns = $I_c = 0.5\, I_b = 0.5$
Radius of curvature for all spans = r = 20 ft 0 in.

The following coefficients were selected from curves that were drawn from the values recorded in Tables 1 and 2 for a value of $k = \dfrac{EI}{GJ} = 1$.

For the end spans 1-2 and 3-4, the following coefficients were obtained:

S_m	= 5.54	S_t	= 1.68
$m_1 l_1$	= 0.175	$t_1 m_1$	= 0.486
m_{12}	= 0.536	t_{12}	= −0.793
$m_1 l_2$	= −0.001	$t_1 m_2$	= 0.018

The fixed-end moments are calculated from C_m and C_t as follows:

$M_{12} = -qr^2 C_m = -(1)\,(20)^2\,(0.042) = -16.8$ ft-kips
$T_{12} = qr^2 C_t = (1)\,(20)^2\,(-0.00036) = -0.15$ ft-kips

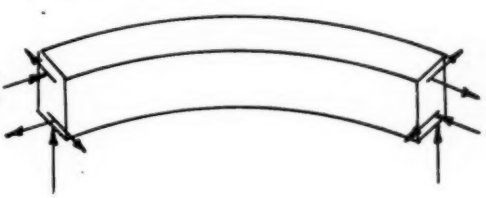

Fig. 3—A sign convention that can be applied to the end of any span

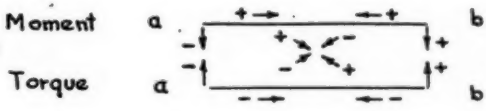

Fig. 4—Proper signs for carry-over factors

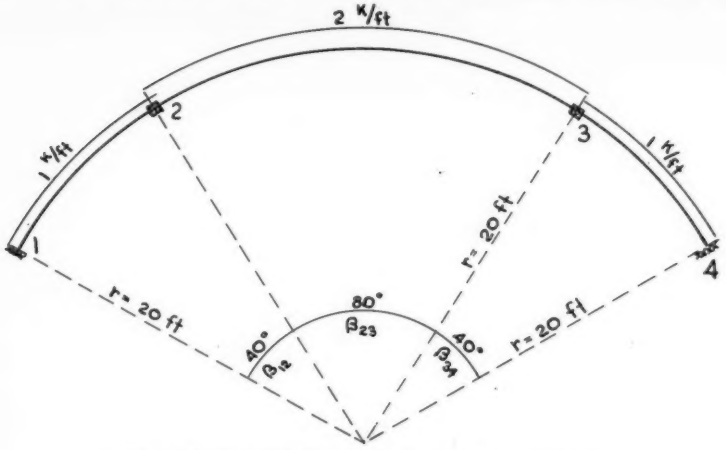

Fig. 5—Example problem: continuous circular curved beam

For the center span 2-3, the coefficients are:

S_m	= 2.34	S_t	= 1.12
m_2l_3	= −0.048	t_3m_3	= 0.105
m_3l_2	= 0.348	t_2m_2	= 0.664
m_{23}	= 0.647	t_{23}	= −0.472

The fixed-end moments for span 2-3 are:

$$M_{23} = -M_{32} = -(2)\,(20)^2\,(0.1777) = -141.5 \text{ ft-kips}$$
$$T_{23} = \quad T_{32} = \quad (2)\,(20)^2\,(-0.0057) = -4.56 \text{ ft-kips}$$

The distribution factors at joints 2 and 3 for bending and torsional couples at the ends of the curved beams and bending couples in the square columns will be calculated. A stiffness coefficient of 3.5 is used for each column corresponding to a restraint of half fixity at the base.

For bending couples at joint 2:

Member	S_m		$\frac{I}{r}$ or $\frac{I}{L}$			Distribution factors
1-2	5.54	×	1/20	=	0.278	0.53
2-3	2.34	×	1/20	=	0.117	0.23
col.	3.5	×	0.5/14	=	0.125	0.24
		Total			0.520	1.00

For torsional couples at joint 2:

Member	S_t		$\frac{I}{r}$ or $\frac{I}{L}$			Distribution factors
1-2	1.68	×	1/20	=	0.084	0.32
2-3	1.12	×	1/20	=	0.056	0.21
col.	3.5	×	0.5/14	=	0.125	0.47
		Total			0.265	1.00

The corresponding values at joint 3 can be obtained from symmetry.

The carry-over factor for the columns for half fixity at the base is $2\,(1 - 0.5)/(4 - 0.5) = 0.286$.

In carrying out the numerical operations shown in Fig. 6, the moments acting on the columns are not recorded as they were not required in the calculations. The distributions and carry-overs for the end moments are shown at the top of Fig. 6 and for the torques at the bottom of the diagram. The distribution for the end moments was first completed as for a straight continuous beam except that all corrections for rotation of the joints are kept in separate columns. This separation is necessary to determine the effect of the rotation upon the torques after all corrections for end moments have been completed. The carry-overs are, of course, placed in the fixed-end moment columns. Thus, when joint 2 is considered, it is found that there is an unbalanced moment of $-141.5 + 16.8$ or -124.7 ft-kips. Therefore corrections of

$$M'_{21} = (0.53)\,(+124.7) = +66.2$$
$$M'_{23} = (0.23)\,(+124.7) = +28.7$$

are recorded in separate columns. The carry-overs to the other ends of the members are

For span 1-2 $\quad (0.536)\,(66.2) = 41.7$
For span 2-3 $\quad (0.647)\,(28.7) = 18.6$

These carry-overs are placed in the same columns as the fixed-end moments.

The unbalanced moment at joint 3 is now $+141.5 + 18.6 - 16.8 = 143.3$ and the corrections are

$$M'_{32} = (0.23)\,(-143.3) = -33.0$$
$$M'_{34} = (0.53)\,(-143.3) = -76.0$$

which are recorded in separate columns. The carry-overs are

For span 3-2 $\quad (0.647)\,(-33.0) = -21.3$
For span 3-4 $\quad (0.356)\,(-76.00) = -41.7$

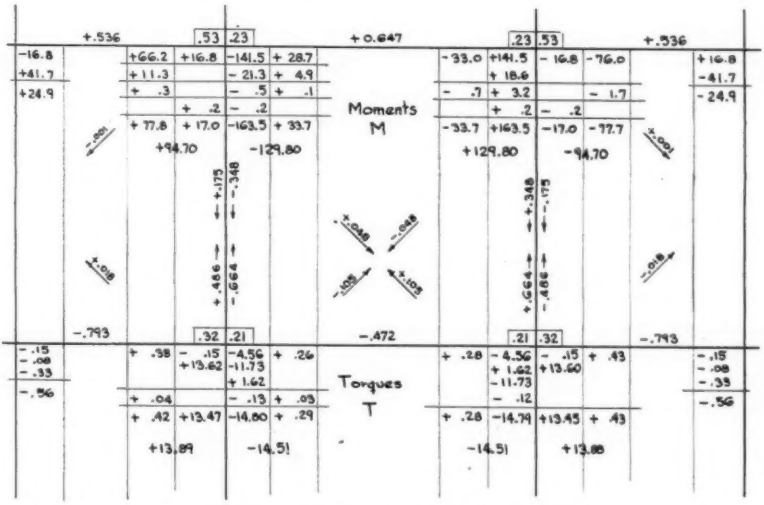

Fig. 6—Example problem: moment distribution method

which are placed in the fixed-end moment columns. Joint 2 is then balanced again and the operations repeated until the corrections can be neglected. After the total corrections for balancing the end moments have been determined, the effect of these changes upon the end torques are determined and added to the original fixed-end torques. The corrections to the end torques for spans 1-2 and 3-4 are

$(0.175)\ (77.8) \quad = +13.62$
$(-0.001)\ (77.8) \quad = -\ 0.08$

for span 2-3

$(-0.348)\ (33.7) \quad = -11.73$
$(-0.043)\ (-33.7) = +\ 1.62$

The end torques are now balanced in a manner similar to the end moments and the corrections at each joint placed in a separate column. The effect of these corrections upon the end moments can now be computed but as they are extremely small they need not be considered. For instance, the change in the end moment at joint 3 for span 2-3 is $(0.664)\ (0.29) - (0.105)\ (0.29) = 0.16$.

The solution is therefore sufficiently accurate and the final end moments and torques are obtained by adding all values at the ends of the members. Thus, the final end moments and torques at the ends of span 1-2 are

$M_{12} = +24.9$ ft-kips
$T_{12} = -0.56$ ft-kips
$M_{2\text{-}1} = \ 77.8 + 17.0 = 94.8$ ft-kips
$T_{2\text{-}1} = \ 13.47 + 0.42 = 13.89$ ft-kips

For span 2-3

$M_{2\text{-}3} = -163.5 + 33.7 = -129.8$
$T_{2\text{-}3} = -14.80 + 0.29 = -14.51$

The moments in the column at joint 2 are:

$129.8 - 94.8 = 35.0$
$14.51 - 13.89 = 0.62$

The moments at the far end of the column are 0.286 times the above values.

SUMMARY

The analysis of continuous beams curved in plan can be made without difficulty by the moment distribution method if both bending and torsional end couples are considered. The necessary fixed-end couples together with the required stiffness and carry-over factors have been presented in this paper. Tabulated values are given from which the necessary coefficients can be selected, but it will be more convenient if diagrams are prepared from the tabulated values.

A proposed method of procedure was illustrated in which the bending end couples were kept separate from the torsional couples. The mathematical operations are not difficult as the convergence is rapid. However, errors due to using wrong signs can easily be made unless the sign convention adopted is definite and easy to apply.

ACKNOWLEDGMENT

The author wishes to express her appreciation to Dr. A. Olivares and J. Otaola for their help in the preliminary work and to Prof. L. C. Maugh of the University of Michigan for his advice and suggestions.

Tests on tile-concrete joists indicate the need for revision of some concepts in the general theory of diagonal tension.

Shear Resistance of Tile-Concrete Floor Joists*

By J. NEILS THOMPSON† and PHIL M. FERGUSON†

SYNOPSIS

Tests on certain types of tile-concrete joists indicate that the tile webs are more effective in resisting diagonal tension than is indicated by the current ACI Building Code specification. Stagger of tile joints appears to be unnecessary, since they do not seem to be plan s of weakness insofar as diagonal tension is concerned. The tile reduces the deflection of the joist.

INTRODUCTION

Precast tile-and-reinforced-concrete floor slabs have been used extensively in Iowa and the Southwest during the past ten years. The low cost of these floor systems, coupled with their fire resistance, makes them highly usable in residential, school and light commercial structures. However, there have been a number of questions about the structural action of composite beams of tile and reinforced concrete. Recent investigations[1,2] under the sponsorship of the Structural Clay Products Institute and the Department of Commerce have studied various phases of these systems to clarify some of these questions. Of two types of tile units studied, a unit called Joistile has proven to be the most used and this report will be devoted to the investigation of floor slabs using this type of tile.

JOISTILE FLOOR CONSTRUCTION

A floor or roof slab of Joistile (Fig. 1) is built [3] in five steps: (1) The "knock-out piece" indicated at "A" on the Joistile detail is broken out by tapping sharply at the center. The Joistile (each 12 in. long) are then laid in line, end to end, on a firm flat surface until the desired length of beam is obtained. This forms a trough running the length of the row of tile. (2) The proper amount of reinforcing steel is placed in this trough and the trough is then filled with concrete and cured. (3) The beams are then placed on supports, either

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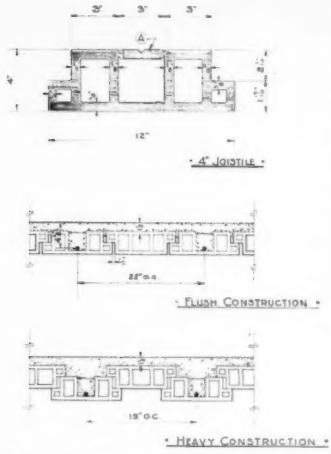

Fig. 1—Floor or roof slab construction of Joistile

bearing walls or girders, and properly spaced for the design to be used. Unless the precast beams are short they will show some deflection. This is corrected by shoring at the center. (4) Filler tile, of the same design as Joistile, without the kerfing that forms the "knock-out piece," is then dropped between the beams (Fig. 2). At this point, although the slab has not yet been placed, the beam strength is ample to carry the construction load. (5) For the final step, top reinforcing is placed, if the design requires it, piping and electrical conduits are set and the whole surface is covered with a topping of concrete to a depth determined by the design. The slab is cured as specified by the design and then shoring is removed.

MATERIALS USED IN TESTS

The concrete used had a cement factor of 4.73 sacks per cu yd and a water factor of 7.67 gal. per sack of cement. Washed river sand and gravel were used with the gravel passing a $\frac{5}{8}$-in. screen. The aggregates and cement were placed in a 3.0 cu ft mixer and mixed dry before adding the water. After the water was added the concrete was then mixed until a uniform mixture was obtained. Standard A.S.T.M. cylinders were made from each batch of concrete. Standard slump tests varied from 2 to 4 in. with an average about $2\frac{3}{4}$ in. This slump would be too low for placing concrete without the use of a mechanical vibrator. For hand tamping the slump should be at least 4 in.

Both cylinders and beams were cured by covering with wet cement sacks for seven days, and were normally tested at 28 days. The compressive strength of these cylinders varied from 2540 to 3520 psi with an average equal to 3150 psi.

The steel was $\frac{3}{4}$-in. round deformed bars of intermediate grade billet steel.

The physical properties of the several tile used in the investigation are given in Table 1. The average absorption was determined by the procedure specified for structural clay tile by A.S.T.M. Designation C 112-36.

The compressive strength of the tile was determined by testing specimens cut from the shoulders of the tile. These specimens were approximately 2 in. square and 4 in. long. They were capped on both ends with plaster of paris to give plane bearing surfaces and then tested in a hydraulic testing machine to failure. The compressive strength was determined by dividing the ultimate load by the net cross-sectional area.

The modulus of rupture was determined by testing specimens cut from the tile having approximate dimensions of 2 x $\frac{5}{8}$ x 12 in. These specimens

were tested using a span of 7 in. and with the load applied at the center of the span through a 1-in. round bar resting on a steel plate ¼ x 1 x 2 in.

The tensile strength of the tile was determined by testing specimens cut from the tile shells. These specimens were similar to the A.S.T.M. cement mortar briquet except the midsection was lengthened to 1 in. Tests were made in a standard A.S.T.M. briquet testing machine.

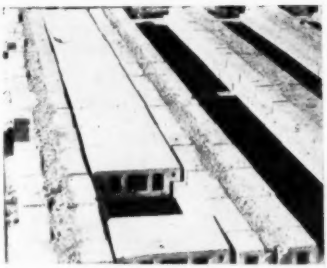

Fig. 2—Top of slab showing the method of placing the filler tile before placing the concrete topping

The modulus of elasticity was determined by attaching four SR-4 electric strain gages to the compression specimens, two on the tension specimens, and taking corresponding stress-strain readings.

DESIGN OF BEAM SPECIMENS

The reinforced Joistile-concrete type of slab construction is in effect a series of T-beams. It is possible to secure two depths of beams depending upon how the filler tile are placed, as shown in Fig. 1. Previous investigations have shown that the general design of these beams should proceed in accordance with ordinary reinforced concrete T-beam theory and that the specifications of the ACI Building Code are applicable. There are several problems in conjunction with this design, however, that made further investigation desirable, namely:

1. The extent to which the tile shells should be counted in estimating resistance to diagonal tension.
2. The effect of the quality of tile upon the diagonal tension resistance of the beam.
3. The adequacy of the bond between the concrete and tile.
4. The deflection of the joistile beam compared to a concrete beam of equal depth and width.

TABLE 1—PHYSICAL PROPERTIES OF TILE

Tile	Description	Absorption,* percent	Compressive strength, psi	Modulus of rupture, psi	Tensile strength, psi	Modulus of elasticity in comp., psi $\times 10^{-6}$	Modulus of elasticity in tension, psi $\times 10^{-6}$
A	Buff color, dense, very uniform, no pits or cracks	5.7	12,440	2630	818	3.0	3.0
B	Red color, light, not very uniform, numerous pits and cracks	12.2	4160	1190	566	2.1	2.7
C	Dark red color, dense uniform, few pits and cracks	12.7	12,110	1210	656	2.6	2.3
D	Medium red color, medium weight, nonuniform, number of small rocks and clay balls in the makeup	8.3	9900	1665	604	3.0	3.2

*One hour boil.

The beams in this test were designed to fail in diagonal tension by: (1) using a relatively short span; (2) providing sufficient steel to prevent failure in tension; (3) providing a flange of sufficient area to prevent failure in compression; (4) providing anchorage for the steel to prevent failure in bond.

The beams were all 8 ft long, for testing on a 7-ft simple span. The reinforcement for each beam was two ¾-in. round deformed steel bars of an intermediate grade welded to a short end anchorage plate (except that no anchorage was used for specimen H-3). The several cross sections used are shown in Fig. 3. Tile from four different plants were used, designated as tiles A, B, C and D. No attempt was made to fill the joints between successive tiles with mortar.

BEAM TESTS

All beams were tested in the same manner (Fig. 4). The load was applied by a calibrated hydraulic jack bearing against a yoke and resting on a steel I-beam which transferred the load to bearings at the third points of the span. These bearings consisted of ¾-in. square steel bars embedded in plaster of paris to give a uniform pressure. Test observations included: (1) deflection at mid-point of the beam; (2) slip between the beam tile and the concrete slab; (3) ultimate load; (4) type of failure. Tables 2 and 3 show the results of the tests.

DISCUSSION OF RESULTS

Resistance to diagonal tension

Since the tile joints in this type of construction occur at regular intervals and since these joints are continuous around three sides of the web, it might be assumed that these joints would create planes of weakness, thus lowering the beam's resistance to diagonal tension. That such is not the case is clearly indicated by the summary of beam tests (Tables 2 and 3). For the shallow sections of Table 2, the three concrete control beams (Series G) failed at $v = 0.066f'_c$, while tile beams (Series H) gave an average $v = 0.099f'_c$. For the deeper sections of Table 3, the concrete control beams (Series N) failed at $v = 0.070f'_c$, while the Joistile-concrete beams (Series K) averaged $v = 0.101f'_c$. This indicates that the reduced vertical shear area at joints is not a matter of any consequence insofar as diagonal tension failure is concerned.

Fig. 3—Cross section of beam specimens

The ACI Building Code (and also the Joint Committee specification) requires that tile joints on

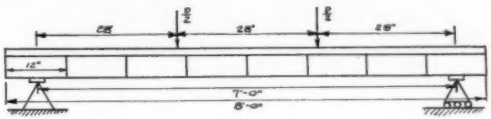

Fig. 4—Elevation showing location of tile joints and loads

opposite sides of the joist web be staggered, that is, if the tile webs in contact with the concrete are to be counted as resisting diagonal tension. These tests indicate no need for specifying stagger.

The actual load capacity of the tile-concrete joists was substantially greater than that of the corresponding concrete control beams. (The web width of the control beams included 1.25 in. of concrete to represent two tile webs, as suggested by the ACI Building Code. See Fig. 5a). The exact factors that lead to this extra strength are not established by these tests. Certainly between tile joints the tiles carry some tension and reduce the steel stress, as evidenced by reduced deflection of the beams under equal loads (Fig. 6). The tile also has a greater compressive strength than the concrete; but its indicated tensile strength, as tested separately, is only slightly if any higher and its modulus of elasticity tends to be lower. In some measure, the extra strength, in the case of the deeper Series K beams, may be due to the extra concrete in the upper part of the web, as discussed below. Whatever the reason for the increased strength in diagonal tension, it is substantial and seems to justify the inclusion of all four tile webs in the effective shear width. The unit shear has been recalculated counting all four tile webs (Fig. 5b) in the stem width and these values are tabulated in the last column of Tables 2 and 3 as "Revised v/f'_c." Shear stresses thus calculated compare reasonably with those of the concrete control beams.

The assistance of the outer webs is probably not possible unless the concrete is in contact with the top of the tile. Earlier tests[1] made on joists using the so-called U-tile, where the concrete contacts only the inner web, definitely showed little stress carried by the outer webs, although they showed some surplus of diagonal tension strength.

Table 1 indicates that tile B was slightly stronger in compression than the concrete ($f'_c = 3150$ psi average) and that the other tiles were much stronger. The excess tile strength seems to add to diagonal tension resistance, the stronger tile increasing this as much as 25 percent.

Table 3 also shows the ultimate strength of special concrete beams (Series L) stepped out to a greater web width above mid-depth, thus following the concrete outline of the Joistile-concrete beams of this depth. These beams consistently tested higher in strength, giving $v = 0.084f'_c$, as compared to $0.070f'_c$ for concrete webs of uniform width. This seems to emphasize the recognized need for further study as to the true nature of the variables defining diagonal tension strength.

Bond of concrete to tile

One group of specimens failed in bond of concrete to tile and that was the Joistile-concrete beams using tile C. These failures were caused by a com-

TABLE 2—JOISTILE-CONCRETE FLUSH CONSTRUCTION
(Diagonal tension failures unless noted otherwise)

Tile	Specimen No.	Ult. L.L., lb	v, psi (including D.L.)	f'_c, psi	v/f'_c **	Revised v/f'_c ††
Concrete control beams, d = 4.5 in., b' = 4.25 in.						
—	G-4	5920	188	3320	0.0566	—
—	G-5	6690	212	3150	0.0673	—
—	G-6	7510	236	3170	0.0744	—
					Ave. 0.066	
Joistile-concrete beams, d = 4.5 in., b' = 3.0 in. concrete plus 1.25 in. tile = 4.25 in.,						b' = 5.50 in.
A	H-6	9900	312	2870	0.1087	0.0840
A	H-9	11,040	347	3030	0.1145	0.0885
A	H-13	10,950	345	3290	0.1050	0.0812
					Ave. 0.109	0.085
B	H-2	8600	273	3270	0.0835	0.0646
B	H-5	8600	273	3150	0.0866	0.0670
B	H-8	8360	266	3020	0.0881	0.0681
					Ave. 0.086	0.067
D	H-14	11,710	366	3320	0.1100	0.0850
D	H-15	10,790	338	3160	0.1070	0.0827
D	H-16	10,180	320	3390	0.0945	0.0730
					Ave. 0.104	0.080
A	R-2*	10,880	343	3230	0.1062	0.0821
A	R-3*	9200	291	3560	0.0817	0.0632
					Ave. 0.094	0.073
Joistile-concrete beams *not* failing in diagonal tension						
A	H-3	8100	260	3200	0.0812†	0.0620
C	H-4	7050	228	3520	0.0648‡	
C	H-11	6800	220	3150	0.0700‡	
C	H-12	6800	220	3120	0.0706‡	
					Ave. 0.068§	
A	R-1*	5060	166	3000	0.0553‡	

*Used mortar for precast part, otherwise same as H specimens.
†Failure in bond to reinforcing steel; no anchorage plate.
‡Failure in bond of concrete to tile.
§Average of H-4, H-11 and H-12 only.
**v/f'_c is calculated from stem width b' which includes the two tile webs in contact with concrete web.
††Revised v/f'_c is calculated from a revised stem width which includes all four vertical tile webs.

bination of a "die skin" that had not been removed from the top of the beam tile in manufacture and a concrete mix in this particular series that was unusually dry. This same tile in heavy construction, where the concrete was considerably deeper over the beam tile, did not give this bond failure. One other failure in bond of concrete to tile occurred (Specimen R-1), which was attributed to failure to wet the tile before pouring the topping. Although readings to measure such slip between tile and concrete were made on many specimens, no significant movement before failure was found in any other specimens. In general, it is believed that if the concrete is workable, and the tile has been wetted beforehand, this type of failure is not likely.

Deflection

Observations were made of the mid-point deflection of all beams and Fig. 6 shows the average deflection-load curves for each group. The tile adds considerably to the stiffness of the beams. Very little difference was apparent due to the different strength tiles. These deflections seem to suggest further that the shells not in contact with the concrete web are acting and are con-

TABLE 3—JOISTILE-CONCRETE HEAVY CONSTRUCTION

(All failures in diagonal tension)

Tile	Specimen No.	Ult. L.L., lb	v, psi (including D.L.)	f'_c, psi	v/f'_c	Revised v/f'_c
Concrete control beams, d = 7 in., b' = 4.25 in.						
—	N-1	10,270	206	3000	0.0687	—
—	N-2	10,360	207	2990	0.0692	—
—	N-3	9260	186	2540	0.0732	
					Ave. 0.070	
Joistile-concrete beams, d = 6.25 in., b' = 3.0 in. concrete plus 1.25 in. tile = 4.25 in.,						b' = 5.50 in.
A	K-2	16,250	364	3140	0.1158	0.0895
A	K-5	14,480	308	2830	0.1088	0.0840
A	K-8	16,920	380	3260	0.1163	0.0900
					Ave. 0.114	0.088
B	K-1	13,180	296	3340	0.0887	0.0686
B	K-4	11,650	260	3350	0.0776	0.0600
B	K-7	14,160	318	3500	0.0907	0.0701
					Ave. 0.086	0.066
C	K-3	12,880	292	3050	0.0958	0.0740
C	K-6	15,910	356	3020	0.1180	0.0912
C	K-9	15,390	346	2970	0.1164	0.0900
					Ave. 0.110	0.085
D	K-10	13,190	297	3490	0.0851	0.0656
D	K-11	14,720	330	3340	0.0988	0.0764
D	K-12	15,640	350	3320	0.1054	0.0815
					Ave. 0.096	0.074
Special concrete beams, d = 6.25 in., b' = 4.25 in., b' (above mid-depth) = 7 in.						
—	L-1	11,740	265	3150	0.0841	—
—	L-2	12,890	288	3280	0.0879	—
—	L-3	11,810	265	3220	0.0825	—
					Ave. 0.084	

*v/f'_c is calculated from stem width b' which includes the two tile webs in contact with concrete slab.
†Revised v/f'_c is calculated from a revised stem width which includes all four vertical tile webs.

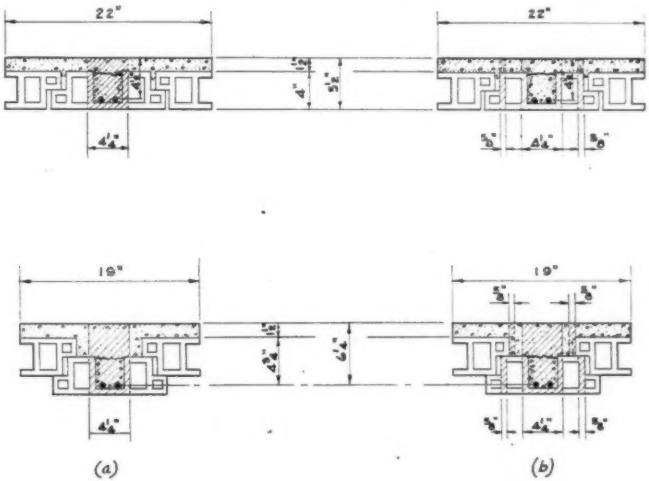

Fig. 5—Shaded areas indicate widths used in: (a) usual v calculations; (b) revised v calculations (last column of Tables 2 and 3)

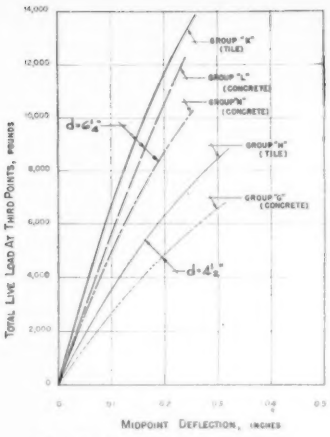

Fig. 6—Load deflection curves

tributing to the stiffness of the beam. It is believed that this is true for this particular tile section, but may not be true for all designs.

CONCLUSIONS

1. With this type of tile, and probably with all types, the presence of open vertical joints between tiles is not a source of weakness insofar as diagonal tension is concerned.

2. It is unnecessary to stagger the tile joints on opposite sides of the web.

3. With this type of construction where concrete covers the top face of the tile, diagonal strength can be based on counting four tile webs per joist.

4. Diagonal tension resistance is increased with high strength tile.

5. The tile reduces the deflection of the joist.

6. Precasting the concrete to the level of the top of the tile is not a source of weakness if care is taken to use workable mixes and to wet the tile before placing the concrete; but care should be exercised in both these matters.

7. Diagonal tension resistance of reinforced concrete beams seems to be increased by added width above mid-depth even when width below mid-depth is unchanged. A 20 percent increase in the unit shear resistance was not expected by the authors.

BIBLIOGRAPHY

1. Thompson, J. Neils and Ramey, W. D., "An Investigation of U-Type Tile-Concrete Beams," *Bulletin* No. 41, Bureau of Engineering Research, University of Texas, Austin, Texas, Jan. 1947.

2. Thompson, J. Neils and Ramey, W. D., "Investigation of Reinforced Joistile-Concrete Beams," *Bulletin* No. 4818, Bureau of Engineering Research, University of Texas, Austin, Texas, May 1948.

3. "Precast Beams of Four-Inch Joistile," Clay Products Association of the Southwest.

4. Giese, Henry and Bridgman, C. T., "Precast Tile Beam Floor," *Bulletin* No. 286, Agriculture Experiment Station, Iowa State College, Ames, Ia., May 1941.

5. Hansen, James H., "Developments in Reinforced Brick Masonry," *Proceedings*, A. S. C. E., Mar. 1933, pp. 407-427.

6. Parsons, D. E. and McBurney, J. W., "Shear Tests of Reinforced Brick Masonry Beams," R. P. 504, U. S. Bureau of Standards, July to Dec. 1932.

7. Wanner, E. F., "Prefabricated Structural Clay Tile Floor System," *Proceedings*, A. S. C. E. V. 20, No. 8, Aug. 1941.

8. Giese, Henry and Bridgman, C. T., "Greater Market Possibilities Promised by Reinforced Clay Tile Floor," *Brick and Clay Record*, June 1935, pp. 201-202.

9. Parsons, D. E. and Stang, A. H., "Test of Composite Beams and Slabs of Hollow Tile and Concrete," R. P. 181, U. S. Bureau of Standards, Jan.-June 1930.

The fundamental basis of finite differences is not involved when stripped of its mathematical mantle and presented as a geometrical relationship.

Solution of Difficult Structural Problems by Finite Differences*

By ALFRED PARME†

SYNOPSIS

Finite differences can be applied to the solution of those structural problems in which the physical relationships are expressed as a differential equation. Essentially, the technique employed consists of replacing the derivatives of the differential equation by its central difference equivalent. The problem is thus reduced to the simple task of solving a system of simultaneous linear algebriac equations. The numerical computation involved in the procedure is considerably reduced by two devices. First, the number of equations necessary to attain sufficient accuracy is reduced by an evaluation of the error introduced in substituting central differences for derivatives. Secondly, the solution of simultaneous equations is speeded by a systematic rapid tabulation of easily determined values.

The procedure is applied to the design of a sheet pile wall, elliptical dome and skewed bridge to illustrate the scope and simplicity of the method.

INTRODUCTION

In the last few years tremendous strides have been made in the understanding of indeterminate structures due principally to introductions of simplifying numerical procedures. In sharp contrast to this progress the analysis of some two and three dimensional structural members such as slabs and shells still remains a mystery to the average structural designer. The lack of progress in this field is primarily because these problems involve the solution of differential equations. For instance, the analysis of a slab requires the solution of the differential equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{EI}$$

in which w is the deflection of the slab and p is the loading on the slab. On the other hand, to determine the bending moments in a circular dome created by concentric loading, the differential equations

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$$\frac{d^2u}{d\phi^2} + \cot\phi \frac{du}{d\phi} - \cot^2\phi\, u - Etr\, v = 0$$

$$\frac{d^2v}{d\phi^2} + \cot\phi \frac{dv}{d\phi} - \cot^2\phi\, v + \frac{ru}{EI} = 0$$

must be solved.

For the most part, text books on these problems treat the subject in a rigorous manner aimed, not so much at the solution of typical problems, but at instructing students in the theory of elasticity. Consequently the mathematics employed, while not formidable to individuals schooled in stress analysis, is nevertheless unfamiliar to the average designer. But such highly involved mathematics is unnecessary. Any type of problem involving differential equations can be solved with no greater complication than the solution of a group of simultaneous equations, arising from a finite difference procedure.

APPLICATION OF FINITE DIFFERENCE EQUATIONS

The application of finite difference equations to the solution of difficult structural problems is in large measure comparable to the technique now used to surmount mathematical difficulties in the solution of indeterminate structures. Generally, due either to variations in the shape and size of the members or irregularities in the loading pattern, the integrations of the moment area curves involve lengthy and difficult mathematical manipulations. To avoid such intricate operations the required integrations of the various functions are approximated by the summation of a number of specific values which represent the average area for a finite segment. This substitution of the numerical procedure of summation for the more precise operation of integration has its counterpart in the replacement of the differential equation by the approximate finite differences equation.

Just as the replacement of an integral by the summation procedure involves the use of average values of the ordinate, so the replacement of a derivative by finite differences is based on taking the difference of average values of the ordinates. In this light then, it is evident from geometrical considerations of Fig. 1 that if $y = f(x)$ then

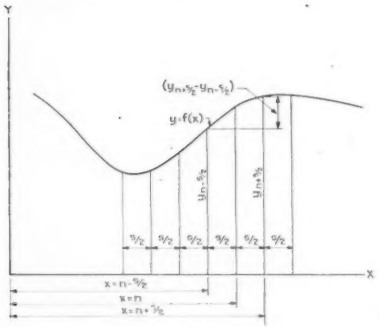


Fig. 1—Geometrical representation of derivative by finite differences

$$\frac{dy}{dx_n} \doteq \frac{y_{n+s/2} - y_{n-s/2}}{s} \quad \text{(1a)}$$

in which $y_{n+s/2}$ represents the ordinate at $x = n + \frac{s}{2}$ and $\doteq$ means approximately equal to. By repeating this process, it naturally follows that

$$\frac{d^2y}{dx^2_n} = \frac{d}{dx}\left(\frac{dy}{dx}\right)_n \doteq \frac{y_{n+s} - y_n}{s^2} - \frac{y_n - y_{n-s}}{s^2} \doteq \frac{y_{n+s} - 2y_n + y_{n-s}}{s^2} \quad \text{(1b)}$$

Following the same procedure, we have

$$\frac{d^3y}{dx^3_n} \doteq \frac{y_{n+3s/2} - 3y_{n+s/2} + 3y_{n-s/2} - y_{n-3s/2}}{s^3} \quad \text{(1c)}$$

and

$$\frac{d^4y}{dx^4_n} \doteq \frac{y_{n+2s} - 4y_{n+s} + 6y_n - 4y_{n-s} + y_{n-2s}}{s^4} \quad \text{(1d)}$$

The finite difference equivalents for the odd order derivatives are given in terms of the ordinate at midpoints, while those for the even order are given by values at whole integers from n. However, to make the approximation usable it is necessary to express the odd order finite difference equations in terms of ordinates at the same points as the even order. To do so, we need only to express the ordinates at the midpoints as the average of the two adjacent values. On this premise we have

$$\frac{dy}{dx_n} \doteq \frac{\frac{y_{n+s} + y_n}{2} - \frac{y_n + y_{n-s}}{2}}{s} \doteq \frac{y_{n+s} - y_{n-s}}{2s} \quad \text{(1e)}$$

and

$$\frac{d^3y}{dx^3_n} \doteq \frac{y_{n+2s} - 2y_{n+s} + 2y_{n-s} - y_{n-2s}}{2\,s^3} \quad \text{(1f)}$$

The relationship just derived is admittedly approximate. The exact relationship between finite differences and derivatives can be obtained by Taylor's theorem which states:

$$f(m) = f(n) + (m-n)\frac{d(n)}{dx} + \frac{(m-n)^2\,d^2(n)}{2!\quad dx^2} + \frac{(m-n)^3\,d^3(n)}{3!\quad dx^3} + \cdots \quad \text{(2)}$$

in which $f(m)$ represents the function of x at $x = m$ and $f(n)$ equals the function of x at $x = n$. Since y is equal to the function of x, then we have

$$y_m = y_n + (m-n)\frac{dy}{dx_n} + \frac{(m-n)^2\,d^2y}{2!\quad dx^2_n} + \frac{(m-n)^3\,d^3y}{3!\quad dx^3_n} + \frac{(m-n)^4\,d^4y}{4!\quad dx^4_n} + \cdots \quad \text{(3)}$$

If we let m assume the successive values of $n + 2s$, $n + s$, n, $n - s$, $n - 2s$ we have

$$y_{n+2s} = y_n + 2s\frac{dy}{dx_n} + \frac{(2s)^2\,d^2y}{2!\quad dx^2_n} + \frac{(2s)^3\,d^3y}{3!\quad dx^3_n} + \frac{(2s)^4\,d^4y}{4!\quad dx^4_n} + \cdots \quad \text{(4a)}$$

$$y_{n+s} = y_n + s\frac{dy}{dx_n} + \frac{s^2\,d^2y}{2!\,dx^2_n} + \frac{s^3\,d^3y}{3!\,dx^3_n} + \frac{s^4\,d^4y}{4!\,dx^4_n} + \cdots \quad \text{(4b)}$$

$$y_n = y_n \quad \text{(4c)}$$

$$y_{n-s} = y_n - s\frac{dy}{dx_n} + \frac{s^2\,d^2y}{2!\,dx^2_n} - \frac{s^3\,d^3y}{3!\,dx^3_n} + \frac{s^4\,d^4y}{4!\,dx^4_n} - \cdots \quad \text{(4d)}$$

$$y_{n-2s} = y_n - 2s\frac{dy}{dx_n} + \frac{(2s)^2\,d^2y}{2!\quad dx^2_n} - \frac{(2s)^3\,d^3y}{3!\quad dx^3_n} + \frac{(2s)^4\,d^4y}{4!\quad dx^4_n} - \cdots \quad \text{(4e)}$$

Adding algebraically the appropriate expressions we find

$$\frac{y_{n+s} - y_{n-s}}{2s} = \frac{dy}{dx_n} + \frac{s^2}{3!}\frac{d^3y}{dx^3_n} + \frac{s^4}{5!}\frac{d^5y}{dx^5_n} + \frac{s^6}{7!}\frac{d^7y}{dx^7_n} + \cdots \quad (5a)$$

$$\frac{y_{n+s} - 2y_n + y_{n-s}}{s^2} = \frac{d^2y}{dx^2_n} + \frac{2s^2}{4!}\frac{d^4y}{dx^4_n} + \frac{2s^4}{6!}\frac{d^6y}{dx^6_n} + \frac{2s^6}{8!}\frac{d^8y}{dx^8_n} + \cdots \quad (5b)$$

$$\frac{y_{n+2s} - 2y_{n+s} + 2y_{n-s} - y_{n-2s}}{2s^3} = \frac{d^3y}{dx^3_n} + s^2\frac{(2^5-2)}{5!}\frac{d^5y}{dx^5_n} + s^4\frac{(2^7-2)}{7!}\frac{d^7y}{dx^7_n} + \cdots \quad (5c)$$

$$\frac{y_{n+2s} - 4y_{n+s} + 6y_n - 4y_{n-s} + y_{n-2s}}{s^4} = \frac{d^4y}{dx^4_n} + s^2\frac{(2^7-8)}{6!}\frac{d^6y}{dx^6_n} +$$

$$s^4\frac{(2^9-8)}{8!}\frac{d^8y}{dx^8_n} + \cdots \quad (5d)$$

The above equations express finite differences as a sum of a series of ascending derivatives. The inverse relationship, that is expressing the derivative in terms of its differences is obtained by eliminating successively the higher order derivatives. If the finite differences be designated by the symbol δ, then successive eliminations leads to the following relationships

$$\frac{dy}{dx_n} = \delta^1 - \frac{s^2\,\delta^3}{6} + \frac{s^4\,\delta^5}{30} - \cdots \quad (6a)$$

$$\frac{d^2y}{dx^2_n} = \delta^2 - \frac{s^2\,\delta^4}{12} + \frac{s^4\,\delta^6}{90} - \cdots \quad (6b)$$

$$\frac{d^3y}{dx^3_n} = \delta^3 - \frac{s^2\,\delta^5}{4} + \frac{7\,s^4\,\delta^7}{120} - \cdots \quad (6c)$$

$$\frac{d^4y}{dx^4_n} = \delta^4 - \frac{s^2\,\delta^6}{6} + \frac{7\,s^4\,\delta^8}{240} - \cdots \quad (6d)$$

These equations show that approximating a derivative by the first term on the right hand side involves an error proportional to the square of the chosen interval. The use of two terms on the right hand side to approximate a derivate involves an error of approximately the fourth power of the interval s. Hence the error introduced by the replacement of a derivative by its finite difference equivalent can be reduced by either decreasing the size of the interval, or by using more terms. In general it would be found that the latter procedure will produce greater accuracy with less labor.

Since a differential equation expresses a functional relationship valid at any point, while a finite difference equation expresses a relation for only one point, the replacement of a differential equation by its finite difference equivalent necessitates the setting down of finite differences equations for each point. In the case of a beam or slab, approximations of the loads, shears and moments at each point are expressed as linear functions of the deflections of neighboring points. Thus the problem is reduced to one of solving a number of simultaneous equations.

SHEET PILING WALL

To illustrate the use of finite difference procedure a sheet piling wall retaining a 13 ft earth fill as shown in Fig. 2 will be analyzed. The differential equation for a beam on an elastic foundation is given by

$$EI\frac{d^4w}{dy^4} + k_y w = p \quad \ldots\ldots\ldots\ldots (7)$$

in which w = displacement of the beam, p = load per unit area at any point, k_y = reaction of soil per unit area for a unit deflection, E = modulus of elasticity and I = moment of inertia. Since the lateral resistance of soil is zero at the surface and maximum at the bottom of wall, k_y will be assumed to vary in like manner. Hence, we shall arbitrarily set $k_y = \frac{ky}{h}$ in which k is the soil modulus. Thus Eq. (7) becomes

$$\frac{d^4w}{dy^4} + \frac{k\,y}{EI\,h}w = \frac{p}{EI} \quad \ldots\ldots\ldots (8a)$$

The values of k and EI are assumed at 500 lb per cu in. and 5 x 10^8 psi, respectively. Replacing the differential equation (Eq. 8a) by its finite difference equivalent, we have for a typical point in Fig. 3

$$\frac{w_{n+2} - 4\,w_{n+1} + 6\,w_n - 4\,w_{n-1} + w_{n-2}}{s^4} + \frac{k}{EI}\frac{y_n}{h}w_n = \frac{p_n}{EI} \quad \ldots\ldots\ldots\ldots (8b)$$

Since s is taken at 24 in. and $p = \left(\frac{33\text{x}13}{12^2} + \frac{83}{12^3}y\right)$ psi, Eq. (8b) becomes

$$w_{n+2} - 4\,w_{n+1} + \left(6 + \frac{24^4 \times 500 \times y_n}{5 \times 10^8 \times h}\right)w_n - 4\,w_{n-1} + w_{n-2} = (2.9792 + 0.04803\,y)\frac{s^4}{EI} \quad \ldots\ldots\ldots\ldots (8c)$$

or

$$w_{n+2} - 4\,w_{n+1} + \left(6 + 0.33178\frac{y_n}{h}\right)w_n - 4\,w_{n-1} + w_{n-2} = (2.9792 + 0.04803y)\frac{s^4}{EI} \quad \ldots\ldots\ldots (8d)$$

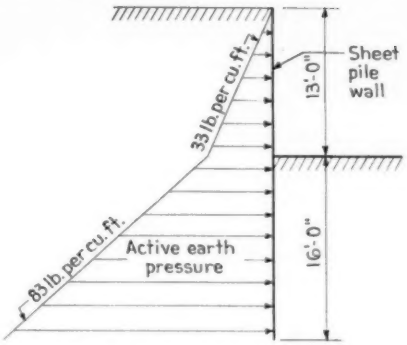


Fig. 2—Diagram of sheet pile wall

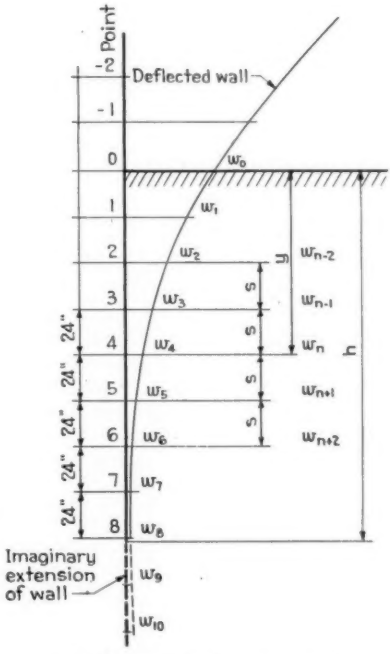


Fig. 3—Location of points on wall

The equations at various points along the piling can be rapidly set down as

$$\left.\begin{aligned}
w_{-2} - 4\,w_{-1} + 6.000\;\; w_0 - 4\,w_1 + w_2 &= 2.9792 \times s^4/EI \\
w_{-1} - 4\,w_0 + 6.0415\,w_1 - 4\,w_2 + w_3 &= 4.1320 \times s^4/EI \\
w_0 - 4\,w_1 + 6.0830\,w_2 - 4\,w_3 + w_4 &= 5.2848 \times s^4/EI \\
w_1 - 4\,w_2 + 6.1244\,w_3 - 4\,w_4 + w_5 &= 6.4375 \times s^4/EI \\
w_2 - 4\,w_3 + 6.1659\,w_4 - 4\,w_5 + w_6 &= 7.5903 \times s^4/EI \\
w_3 - 4\,w_4 + 6.2074\,w_5 - 4\,w_6 + w_7 &= 8.7431 \times s^4/EI \\
w_4 - 4\,w_5 + 6.2488\,w_6 - 4\,w_7 + w_8 &= 9.8959 \times s^4/EI \\
w_5 - 4\,w_6 + 6.2903\,w_7 - 4\,w_8 + w_9 &= 11.049 \times s^4/EI \\
w_6 - 4\,w_7 + 6.3318\,w_8 - 4\,w_9 + w_{10} &= 12.201 \times s^4/EI
\end{aligned}\right\} \quad \ldots\ldots (9)$$

It should be noted that in the above group there are thirteen unknowns and only nine equations. To eliminate four of the unknowns, the end conditions must be introduced. At $y = 0$ the moment and shear produced by the earth retained above $y = 0$ are

$$M = -\frac{33 \times 13^3}{6} = -12{,}080 \text{ in.-lb.}$$

$$V = -\frac{33 \times 13^2}{2 \times 12} = -232.4 \text{ lb.}$$

with

$$M = -EI\frac{d^2w}{dx^2} \doteq -EI\left(\frac{w_{n+1} - 2\,w_n + w_{n-1}}{s^2}\right) \quad \ldots\ldots (10a)$$

$$V = -EI\frac{d^3w}{dx^3} \doteq -EI\left(\frac{w_{n+2} - 2\,w_{n+1} + 2\,w_{n-1} - w_{n-2}}{2\,s^3}\right) \quad \ldots\ldots (10b)$$

then at $y = 0$

$$w_1 - 2\,w_0 + w_{-1} = \frac{12{,}080}{s^2}\frac{s^4}{EI} = 20.98\frac{s^4}{EI} \quad \ldots\ldots (10c)$$

$$w_2 - 2\,w_1 + 2\,w_{-1} - w_{-2} = \frac{232.4 \times 2}{s}\frac{s^4}{EI} = 19.37\frac{s^4}{EI} \quad \ldots\ldots (10d)$$

from which,

$$w_{-1} = 2\,w_0 - w_1 + 20.98\frac{s^4}{EI} \quad \ldots\ldots (10e)$$

$$w_{-2} = 4\,w_0 - 4\,w_1 + w_2 + 22.59\frac{s^4}{EI} \quad \ldots\ldots (10f)$$

$$w_{-2} - 4\,w_{-1} = -4\,w_0 + w_2 - 61.32\frac{s^4}{EI} \quad \ldots\ldots (10g)$$

At the bottom edge, since the moment and shear is zero, it is apparent from the preceding expressions that

$$w_9 = 2\,w_8 - w_7 \quad \ldots\ldots (10h)$$

$$w_{10} - 4\,w_9 = -4\,w_8 + w_6 \quad \ldots\ldots (10i)$$

Thus the unknowns w_{-2}, w_{-1}, w_9, and w_{10} can be eliminated immediately in Eq. (9). The coefficients of the w's in the resulting equations are tabulated in Table 1 in the appropriate row and column.

Indirect solution

The above equations can be solved by either an indirect or a direct solution. In the indirect solution, trial values based either on experience or on temporarily imposed conditions are assumed initially. These assumed values are

TABLE 1—SIMULTANEOUS EQUATIONS FOR SHEET PILING WALL

Row	Col. 0	1	2	3	4	5	6	7	8	9
	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	
0	+2	−4	+2							= 64.3 s^4/EI
1	−2	5.042	−4	+1						= −18.5 s^4/EI
2	+1	−4	6.083	−4	+1					= 5.28 s^4/EI
3		1	−4	6.124	−4	1				= 6.44 s^4/EI
4			1	−4	6.166	−4	1			= 7.59 s^4/EI
5				1	−4	6.207	−4	1		= 8.74 s^4/EI
6					1	−4	6.249	−4	1	= 9.90 s^4/EI
7						1	−4	5.290	−2	= 11.0 s^4/EI
8							2	−4	2.332	= 12.2 s^4/EI

subsequently modified so as to approach the correct values by repeated applications of converging increments. The method of moment distribution is a familiar example of this technique. Other widely used variations of this technique are the iteration process and Southwell's relaxation procedure.

Direct solution—Crout method

On the other hand, the direct solution employs a step by step procedure in which the unknowns are successively eliminated. In general, it has been found that a direct solution is faster for most problems arising in structural design. One of the more rapid methods recently developed to solve simultaneous equations by machine operations is that by Prescott D. Crout.*

In this method, the original simultaneous equations are transformed into a second set of equivalent equations in which all unknowns to the left of a diagonal drawn through the original simultaneous equations are eliminated. The unknowns to the left of the diagonal are eliminated by expressing them in terms of the unknowns remaining on the right side of the diagonal. However, instead of writing each equation and eliminating, as in the ordinary procedure, the labor is systematized by calculating an auxiliary set of values which are tabulated to the left of the diagonal as shown in Table 2, while values to the right of the diagonal represent coefficients of the second set of simultaneous equations mentioned previously. If the coefficients of the original set of simultaneous equations are designated as A_{ij}, in which the subscript ij indicates the row and column respectively in which the coefficients occur, while the coefficients of the second set are designated as B_{ij}, then the elements of the auxiliary system, designated as C_{ij} are given by

$$C_{ii} = A_{ii} - \sum_{k=0}^{k=i-1} C_{ik} B_{ki} \quad \text{(11a)}$$

$$C_{ij} = A_{ij} - \sum_{k=0}^{k=j-1} C_{ik} B_{kj} \quad \text{(11b)}$$

*"A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," *Transactions*, A.I.E.E., V. 60, 1941.

TABLE 2—SOLUTION OF EQUATIONS FOR SHEET PILING WALL

Row	Col. 0	1	2	3	4	5	6	7	8	9
	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	
0	2 | 1	−2	+1							= 32.15 s^4/EI
1	−2	1.042 | 1	−1.920	+.9602						= 44.02 s^4/EI
2	1	−2	1.248 | 1	−1.675	+.8050					= 49.24 s^4/EI
3		1	−2.080	1.681 | 1	−1.384	+.5948				= 38.56 s^4/EI
4			1	−2.325	2.144 | 1	−1.221	+.4664			= 22.38 s^4/EI
5				1	−2.617	2.419 | 1	−1.149	+.4134		= 11.89 s^4/EI
6					1	−2.780	2.589 | 1	−1.101	+.3863	= 7.94 s^4/EI
7						1	−2.851	1.737 | 1	−.5173	= 12.55 s^4/EI
8							2	−1.797	.6293 | 1	= 29.99 s^4/EI
9	395.2	259.2	155.3	86.53	48.27	31.62	27.27	28.07	29.99	w_n
10	−21.0	−32.2	−35.1	−30.5	−21.6	−12.3	−5.2	−1.1	9	$-(w_{n+1}-2w_n+w_{n-1})$
11	0	1550	1850	1550	1150	940	980	1170	1430	$w_n \times 500 \times 144 \times y/h$
12	−12.1	−18.5	−20.2	−17.6	−12.4	−7.1	−3.0	0.6	0	Moment
13	.262	.172	.103	.057	.032	.021	.018	.019	.019	w_n, in.

in which the summation of the product $C_{ik}\,B_{ki}$ or $C_{ik}\,B_{kj}$ is taken from $k = 0$ to $k = i - 1$, or $k = j - 1$. The value of the coefficients for the reduced simultaneous equations is then given by

$$B_{ij} = (A_{ij} - \sum_{k=0}^{k=i-1} C_{ik}\,B_{kj}) \div C_{ii} \qquad (11c)$$

The second set of simultaneous equations shown in Table 2 has been calculated by substituting numerical values in Eq. (11a), (11b) and (11c). Since the various coefficients are dependent on values of the preceding rows and columns, it is necessary to proceed from the upper left hand corner. In this manner, the elements of the first column are calculated first. Then the elements of the first row to the right of the first column is calculated. Following this the second column below the first row is calculated. Next the second row to the right of the second column is determined. This process is repeated for the remaining columns and rows. By this process the value of the unknown in the last column is determined. With this one value determined, the other unknowns are evaluated by successive substitution in the reduced equations.

To illustrate the procedure, the coefficient in row 4, column 3 will be calculated. By Eq. (11b) we have

$$\begin{aligned} C_{ij} = C_{4,3} &= A_{4,3} - (C_{4,0} \times B_{0,3} + C_{4,1} \times B_{1,3} \times C_{4,2} \times B_{2,3}) \\ &= -4 - (0 \times 0 + 0 \times 0.9602 + 1 \times (-) 1.675) \\ &= -2.325 \end{aligned}$$

Similarly, the value for the diagonal C_{44} by Eq. (11a) is

$$\begin{aligned} C_{4,4} &= A_{4,4} - (C_{4,0} \times B_{0,4} + C_{4,1} \times B_{1,4} + C_{4,2} \times B_{2,4} + C_{4,3} \times B_{3,4}) \\ &= 6.166 - (0 \times 0 + 0 \times 0 + 1 \times 0.8050 + (-) 2.325 \times (-) 1.384 \\ &= 2.144 \end{aligned}$$

Using the relationship expressed in Eq. (11c) we have as the coefficient in row 4 column 5

$$B_{ij} = B_{4,5} = [A_{4,5} - (B_{4,0} \times B_{0,5} + C_{4,1} \times B_{1,5} + C_{4,2} \times B_{2,5} + C_{4,3} \times B_{3,5})] \div C_{4,4}$$
$$= [-4 - (0 \times 0 + 0 \times 0 + 1 \times 0 + (-) 2.325 \times 0.5948)] \div 2.144$$
$$= -1.221$$

Table 2 has been evaluated by the procedure outlined above. It can be seen that the value of w_8 is given in row 8. With this unknown thus determined, the remaining w's can be readily evaluated by inserting progressively the calculated w's in the next higher equation in Table 2. In row 9 of Table 2 the displacement of the wall in terms of s^4/EI at each point is tabulated. By inserting these values in Eq. (10a) the moment at the various points can be evaluated. The various values of the soil reaction in lb per sq ft and moment in ft-kips are tabulated in row 11 and 12, from which the curves in Fig. 4 are drawn. The distribution of soil reaction shown in Fig. 4 is considerably different than that given in the conventional method of designing sheet pile walls.

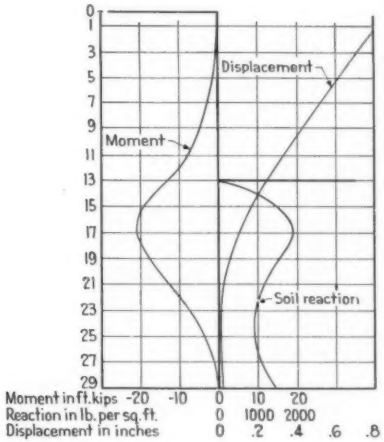


Fig. 4—Variation in moment, displacement and soil reaction

Accuracy of approximations

Accuracy of the approximations can be improved in one of three ways. First, a greater number of points can be used, in other words decreasing the interval s. The second method consists of incorporating the second term on the right hand side of Eq. (6a) to (6d). The third method is by extrapolating from values obtained with fewer points. In most cases the latter two methods will be preferred. For this example the method of extrapolation will be illustrated. Using only the even numbered points shown in Fig. 3 we find that the displacement, bending moment and soil reaction are those given in Table 3.

It was shown that the error introduced by approximating the derivative by the first term of the finite difference series is approximately equal to the

TABLE 3—MOMENT, DISPLACEMENT AND SOIL REACTIONS USING FOUR INTERVALS

Point	0	2	4	6	8
Reaction	0	2310	1240	900	1250
Moment	−12.1	−26.7	−16.5	−4.0	0
Displacement	0.345	0.128	0.034	0.017	0.017

square of the interval. If we designate the error with eight intervals as e_1 and that with four intervals as e_2 then

$$e_1 = C\left(\frac{h}{8}\right)^2 \quad \text{(12a)}$$

$$e_2 = C\left(\frac{h}{4}\right)^2 \quad \text{(12b)}$$

Subracting Eq. (12b) from (12a) we have

$$e_1 - e_2 = C\left[\left(\frac{h}{8}\right)^2 - \left(\frac{h}{4}\right)^2\right] \quad \text{(12c)}$$

Eliminating C in Eq. (12a) we have

$$e_1 = \frac{(e_1 - e_2)\,(\frac{1}{8})^2}{(\frac{1}{8})^2 - (\frac{1}{4})^2} = -\frac{3}{64}(e_1 - e_2) \quad \text{(12d)}$$

Now $(e_1 - e_2)$ is equal to the difference in values obtained with four points and eight points. Therefore, the magnitude of the error can be found at various points. The improved value is equal to the original value minus the calculated error. For example, at point 2 the improved moment value is

$$M_2 = -20.2 - (-)\frac{3}{64}(-20.2 + 26.7) = -20.2 + 2.2 = -18 \text{ ft-kips}$$

Improved values can be obtained in like manner at the other even numbered points. The correction to be applied to the odd numbered points can be obtained by interpolating the corrections from the even numbered points. The moment at point 2, using 16 points, is 17 ft-kips.

ELLIPTICAL DOME

Finite differences procedure is applicable with equal facility to problems involving two variables. To illustrate its application to this type of problem, the bending moments induced by a 45° temperature rise in the elliptical dome of uniform thickness, as shown in Fig. 5, will be determined by finite differences. The behavior of a shell formed by revolving a surface around an axis of rotation is expressed by the two differential equations*

$$\frac{d^2u}{d\phi^2} + \frac{r_1}{r_2}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\cot\phi - \frac{r_2}{r_1}\frac{dt}{d\phi}\right]\frac{du}{d\phi} - \frac{r_1}{r_2}\left(\frac{r_1}{r_2}\cot^2\phi\right)u - \frac{Et\,r_1^2\,V}{r_2} = 0 \quad \text{(13a)}$$

$$\frac{d^2V}{d\phi^2} + \frac{r_1}{r_2}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\cot\phi - \frac{r_2}{r_1}\frac{dt}{d\phi}\right]\frac{dV}{d\phi} - \frac{r_1}{r_2}\left(\frac{r_1}{r_2}\cot^2\phi\right)V + \frac{12r_1^2}{E\,t^2r_2}u = 0 \quad \text{(13b)}$$

*Timoshenko, *Theory of Plates and Shells*, pp. 456 and 457.

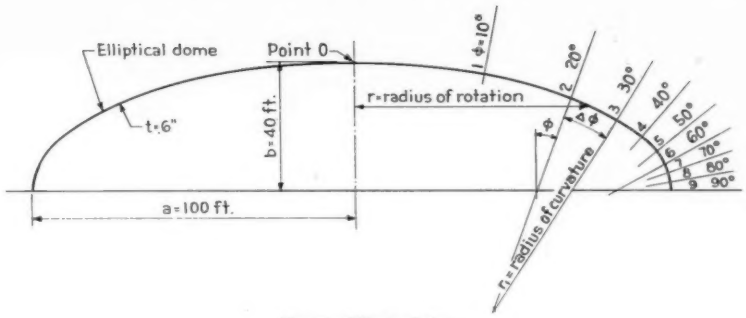


Fig. 5—Elliptical dome

in which

V = angle of rotation in the ϕ direction
u = radial shear multiplied by r_2
r_1 = radius of curvature in the ϕ direction
r_2 = radius of rotation divided by the sin ϕ
t = thickness of shell at any point

For an ellipse,

$$r_1 = \frac{a^2}{b}\frac{1}{(k\sin^2\phi + 1)^{3/2}} \quad \text{(14a)}$$

$$r = \frac{a^2}{b}\frac{\sin\phi}{(k\sin^2\phi + 1)^{1/2}} \quad \text{(14b)}$$

$$r_2 = \frac{a^2}{b}\frac{1}{(k\sin^2\phi + 1)^{1/2}} \quad \text{(14c)}$$

with $k = \frac{a^2}{b^2} - 1$ (14d)

Hence

$$\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) = \frac{d}{d\phi}(k\sin^2\phi + 1) = 2k\sin\phi\cos\phi \quad \text{(14e)}$$

Substituting these expressions in Eq. (13a) and (13b), and canceling the term $\frac{r_2\,dt}{r_1\,d\phi}$, we have

$$\frac{1}{E}\frac{d^2u}{d\phi^2} + \left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi + 1} + \cot\phi\right)\frac{1}{E}\frac{du}{d\phi} - \left(\frac{\cot\phi}{k\sin^2\phi + 1}\right)^2\frac{u}{E} - \frac{a^2t\,V}{b(k\sin^2\phi + 1)^{5/2}} = 0 \quad \text{(15a)}$$

$$\frac{d^2V}{d\phi^2} + \left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi + 1} + \cot\phi\right)\frac{dV}{d\phi} - \left(\frac{\cot\phi}{k\sin^2\phi + 1}\right)^2 V + \frac{12\,a^2u}{Et^3\,b\,(k\sin^2\phi + 1)^{5/2}} = 0 \quad \text{(15b)}$$

The finite difference equations will therefore be

$$\frac{u_{n+1} - 2u_n + u_{n-1}}{E\,(\Delta\phi)^2} + \left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi + 1} + \cot\phi\right)\left(\frac{u_{n+1} - u_{n-1}}{2E\,\Delta\phi}\right) - \left(\frac{\cot\phi}{k\sin^2\phi + 1}\right)^2\frac{u_n}{E} - \frac{a^2t\,V_n}{b\,(k\sin^2\phi + 1)^{5/2}} = 0 \quad \text{(16a)}$$

$$\frac{V_{n+1} - 2V_n + V_{n-1}}{(\Delta\phi)^2} + \left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi + 1} + \cot\phi\right)\left(\frac{V_{n+1} - V_{n-1}}{2\,\Delta\phi}\right) - \left(\frac{\cot\phi}{k\sin^2\phi + 1}\right)^2 V_n +$$

$$\frac{12\,a^2\,u_n}{Et^2b\,(k\sin^2\phi+1)^{5/2}}=0 \qquad (16b)$$

Collecting similar terms of u's and V's

$$\frac{u_{n-1}}{E}\left[\frac{1}{(\Delta\phi)^2}-\frac{1}{2\Delta\phi}\left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi+1}+\cot\phi\right)\right]-\frac{u_n}{E}\left(\frac{2}{(\Delta\phi)^2}+\frac{\cot\phi}{k\sin^2\phi+1}\right)+$$
$$\frac{u_{n+1}}{E}\left[\frac{1}{(\Delta\phi)^2}+\frac{1}{2\Delta\phi}\left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi+1}+\cot\phi\right)\right]-\frac{a^2t}{b}\frac{V_n}{(k\sin^2\phi+1)^{5/2}}=0 \qquad (16c)$$

$$V_{n-1}\left[\frac{1}{(\Delta\phi)^2}-\frac{1}{2\Delta\phi}\left(\frac{2k\sin\phi\cos\phi}{k\sin^2\phi+1}+\cot\phi\right)\right]-V_n\left(\frac{2}{(\Delta\phi)^2}+\frac{\cot\phi}{k\sin^2\phi+1}\right)+$$
$$V_{n+1}\left[\frac{1}{(\Delta\phi)^2}+\frac{1}{2\Delta\phi}\left(\frac{2\,k\sin\phi\cos\phi}{k\sin^2\phi+1}+\cot\phi\right)\right]+\frac{12a^2u_n}{Ebt^3\,(k\sin^2\phi+1)^{5/2}}=0 \qquad (16d)$$

Since the trigonometric functions for the various values of ϕ can be readily evaluated as shown in Table 4, two difference equations for each point can be written. The resulting equations are tabulated in Table 5 from coefficients contained in Table 4. It should be noted that because of symmetry, u and V are zero at $\phi = 0$. It follows that the equations at $\phi = 0$ are superfluous. At the lower edge the equations have been modified to provide for a displacement due to a 45° temperature rise. The radial displacement $\Delta = -0.000006 \times 45 \times 100 = -0.027$ ft but at the boundary with $\sin\phi = 1$, we have

$$\Delta=-\frac{1}{Et}\frac{du}{d\phi}\doteq-\frac{1}{Et}\frac{u_{10}-u_8}{2\Delta\phi} \qquad (17a)$$

Hence

$$\frac{u_{10}}{E}=\frac{u_8}{E}+(0.027t)\,2(\Delta\phi)=\frac{u_8}{E}+4.7124\times10^{-3} \qquad (17b)$$

TABLE 4—COEFFICIENTS FOR ELLIPTICAL DOME

Item	ϕ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
1	$\sin\phi$	0	.17365	.34202	.50000	.64278	.76604	.86603	.93969	.98481	1.0000
2	$\cos\phi$	1	.98481	.93969	.86603	.76604	.64279	.50000	.34202	.17365	0
3	$\cot\phi$	∞	5.6713	2.7475	1.7321	1.1918	.83910	.57735	.36397	.17633	0
4	$\sin^2\phi$	0	.03015	.11698	.25000	.41372	.58682	.75000	.88302	.96985	1.0000
5	$\cos\phi\sin\phi$	0	.17101	.32139	.43301	.49240	.43301	.32139	.32139	.17101	0
6	$k\sin^2\phi+1$	1	1.1583	1.6141	2.3125	3.1720	4.0808	4.9375	5.6359	6.0917	6.25
7	$(k\sin^2\phi+1)^{1/2}$	1	1.0762	1.2705	1.5207	1.7810	2.0201	2.2220	2.3739	2.4681	2.50
8	$(k\sin^2\phi+1)^{3/2}$	1	1.2466	2.0507	3.5166	5.6493	8.2437	10.971	13.380	15.035	15.625
9	$(k\sin^2\phi+1)^{5/2}$	1	1.4440	3.3102	8.1321	17.920	33.641	54.171	75.405	91.589	97.656
10	$(2k\sin\phi\cos\phi\div(k\sin^2\phi+1))+\cot\phi$	∞	7.2215	4.8382	3.6982	2.8217	2.1061	1.4982	.96275	.47109	0
11	$(\cot\phi\div(k\sin^2\phi+1))^2$	∞	23.973	2.8973	.56099	.14116	.04228	.01467	.00417	.00084	0
12	$1\div(\Delta\phi)^2-$ item 10 $\div2\Delta\phi$	∞		18.967	22.233	24.744	26.794	28.536	30.070	31.479	32.828
13	$2\div(\Delta\phi)^2+$ item 11	∞	89.627	68.553	66.217	65.797	65.698	65.670	65.660	65.657	65.657
14	$1\div(\Delta\phi)^2+$ item 10 $\div2\Delta\phi$	∞	53.517	46.689	43.423	40.912	38.862	37.120	35.586	34.178	32.828
15	125 $\div$ item 9	125	86.566	37.763	15.371	6.8798	3.7157	2.3075	1.6578	1.3648	1.2800
16	24,000 $\div$ item 9	24,000	16621	7250.4	2951.3	1339.3	713.41	443.04	318.28	262.03	245.76
17	$r_1 = 250 \div$ item 8	250	200.5	121.9	71.09	44.25	30.32	22.78	18.69	16.62	16.00

Moreover since

$$M = -\frac{Et^3}{12}\frac{1}{r_1}\frac{dV}{d\phi} \doteq -\frac{Et^3}{12} \times \frac{V_{n+1} - V_{n-1}}{r_1\, 2(\Delta\phi)} \quad \text{(18a)}$$

then assuming zero moment at lower edge

$$V_{10} = V_8 \quad \text{(18b)}$$

At point 9, ($\phi = 90°$) the finite difference equations representing Eq. (16c) and (16d) are

$$32.828\,\frac{u_8}{E} - 65.656\,\frac{u_9}{E} + 32.828\,\frac{u_{10}}{E} - 1.2800\,V_9 = 0 \quad \text{(18c)}$$

$$32.828\,V_8 - 65.656\,V_9 + 32.828\,V_{10} + 245.76\,\frac{u_9}{E} = 0 \quad \text{(18d)}$$

Eliminating u_{10} and V_{10} in Eq. (18c) and (18d) by means of Eq. (17b) and (18b) we have for point 9,

$$65.656\,\frac{u_8}{E} - 65.656\,\frac{u_9}{E} - 1.2800\,V_9 = -4.7124 \times 10^{-3} \times 32.828 = -0.15470$$

$$65.656\,V_8 - 65.656\,V_9 - 1.2800\,\frac{u_9}{E} = 0$$

The eighteen simultaneous equations are solved in Table 6. The resulting values of u and V are tabulated in row 19. From these the value of the bending moment is determined by means of Eq. (18a). Intermediate values at $\phi = 85°$ and $75°$ have been obtained by taking the difference of adjacent values. The values are listed in row (21), and sketched in Fig. 6.

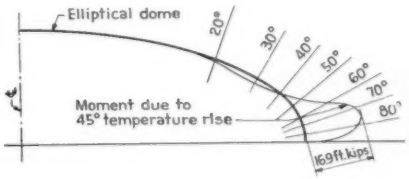


Fig. 6—Bending moment in elliptical dome due to 45° temperature rise

The number of simultaneous equations can be reduced by half by eliminating at the start one of the variables. Thus, the u's in Eq. (16c) can be eliminated by expressing u in terms of V from the relationships obtained by Eq. (16a) at points $n - 1$, n, $n + 1$. While this alternate procedure reduces the number of equations, the expressions become more complex, so that little saving in time is achieved. Which method is preferable will depend to a large extent on the nature of the problem, and what values are desired.

SKEW PLATES

The difference equations obtained for one dimensional analysis can be used to obtain expressions for partial derivatives arising in two dimensional analysis such as plates. By holding one of the derivatives constant, the partial derivatives can be expressed as an operation of one finite difference on the other. If points of a network are located by $m + i$ in the y direction and by $n + j$ in the x direction, then

TABLE 5—EQUATIONS

Col.		1	2	3	4	5	6	7	8	9
Pt.	Row	u_1/E	V_1	u_2/E	V_2	u_3/E	V_3	u_4/E	V_4	u_5/E
1	1	−89.629	−86.566	+53.517						
	2	−16.621	+89.629		−53.517					
2	3	−18.967		+68.553	+37.763	−46.689				
	4		−18.967	−7250.4	+68.553		−46.689			
3	5			−22.233		+66.217	+15.371	−43.423		
	6				−22.233	−2951.3	+66.217		−43.423	
4	7					−24.744		+65.797	+6.8798	−40.912
	8						−24.744	−1339.3	+65.797	
5	9							−26.794		+65.698
	10								−26.794	−713.41
6	11									−28.536
	12									
7	13									
	14									
8	15									
	16									
9	17									
	18									

TABLE 6—SOLUTION OF EQUA-

Row	Col. 1	2	3	4	5	6	7	8	9
1	89.629|1	+.96583	−.59709						
2	−16621	16143|1	−.61479	.00332					
3	−18.967	+18.319	68.490|1	+.55224	−.68169				
4		−18.967	−7262.1	4078.9|1	−1.2137	−.01145			
5			−22.233	+12.278	65.963|1	+.23516	−.65830		
6				−22.233	−2978.3	766.33|1	−2.5584	−.05666	
7					−24.744	+5818.8	64.395|1	+.11224	−.63533
8						−24.744	−1402.6	221.82|1	−4.0173
9							−26.794	+3.0073	60.751|1
10								−26.794	−821.05
11									−28.536
12									
13									
14									
15									
16									
17									
18									
19	0	0	−.003	−.003	−.007	+.454	+.152	+1.166	+.2593
20		0		+.46		+1.17		+.22	
21		0		−.1		−.2		0	

FOR ELLIPTICAL DOME

10	11	12	13	14	15	16	17	18	
V_5	u_6/E	V_6	u_7/E	V_7	u_8/E	V_8	u_9/E	V_9	
									= 0
									= 0
									= 0
									= 0
									= 0
									= 0
									= 0
−40.912									= 0
+3.7157	−38.862								= 0
+65.698		−38.862							= 0
	+65.670	+2.3075	−37.120						= 0
−28.536	−443.04	+65.670		−37.120					= 0
	−30.070		+65.660	+1.6578	−35.586				= 0
		−30.070	−318.28	+65.660		−35.586			= 0
			−31.479		+65.657	+1.3648	−34.178		= 0
				−31.479	−262.03	+65.657		−34.178	
					−65.656		+65.656	+1.2800	= − .15470
						−65.656	−245.76	+65.656	= 0

TIONS FOR ELLIPTICAL DOME

10	11	12	13	14	15	16	17	18	
									= 0
									= 0
									= 0
									= 0
									= 0
									= 0
									= 0
−.18444									= 0
+.07029	−.63963								= 0
118.47 | 1	−4.4331	−.32804							= 0
+2.0057	56.308 | 1	+.05266	−.65923						= 0
−28.536	−569.54	86.302 | 1	−4.3505	−.43012					= 0
	−30.070	+1.5835	52.727 | 1	+.04436	−.67492				= 0
		−30.070	−449.10	72.648 | 1	−4.1722	−.48984			= 0
			−31.479	+1.3964	50.237 | 1	+.04078	−.68032		= 0
				−31.479	−393.37	66.280 | 1	−4.0376	−.51565	= 0
					−65.656	+2.6777	31.800 | 1	.08367	= − 4.8648 × 10^{-2}
						−65.656	−510.86	74.544 | 1	= − 33.339 × 10^{-2}
+.678	+.4796	−4.419	+.3748	−14.065	−.36901	−25.570	−2.0752	−33.339	(× 10^{-2})
−5.58		−14.74		−21.15		−19.27		0	$V_{n+1} - V_{n-1}$
+2.4		+8.3	+12.1	+14.6	+16.9*	+14.7	+12.3*	0	M_n, ft-kips

* Interpolated values.

$$\frac{\partial^2 w}{\partial y \partial x_{mn}} = \frac{d}{dy_m}\left(\frac{dw}{dx_n}\right) \doteq \frac{d}{dy_m}\left(\frac{w_{m+i,\,n+1} - w_{m+i,\,n-1}}{2rs}\right) \quad \text{(19a)}$$

$$\doteq \frac{\dfrac{w_{m+1,\,n+1} - w_{m+1,\,n-1}}{2rs} - \dfrac{w_{m-1,\,n+1} - w_{m-1,\,n-1}}{2rs}}{2s} \quad \text{(19b)}$$

$$\doteq \frac{w_{m+1,\,n+1} - w_{m+1,\,n-1} - w_{m-1,\,n+1} + w_{m-1,\,n-1}}{4rs^2} \quad \text{(19c)}$$

In a similar manner, for $\dfrac{\partial^4 w}{\partial x^2 \partial y^2}$ we have

$$\frac{\partial^4 w}{\partial y^2 \partial x^2{}_{m\,n}} = \frac{d^2}{dy^2{}_m}\left(\frac{d^2 w}{dx^2{}_n}\right) \doteq \frac{d^2}{dy^2{}_m}\left(\frac{w_{m+i,\,n+1} - 2w_{m+i,\,n} + w_{m+i,\,n-1}}{r^2 s^2}\right) \quad \text{(20a)}$$

$$\doteq (w_{m+1,\,n+1} - 2w_{m+1,\,n} + w_{m+1,\,n-1} - 2w_{m,\,n+1} + 4w_{m,\,n} - 2w_{m,\,n-1} +$$
$$w_{m-1,\,n+1} - 2w_{m-1,\,n} + w_{m-1,\,n-1}) \div r^2 s^4 \quad \text{(20b)}$$

The finite difference equivalents for any other partial derivative can be obtained by the same procedure.

One of the problems that can be solved more readily by finite differences than by other methods is that of skew plates. To illustrate the procedure used in two dimensions, a skew plate subject to a uniform load and free along two boundaries while fixed at the other boundaries with the dimensions and angle shown in Fig. 7 will be analyzed. The differential equation for a skew

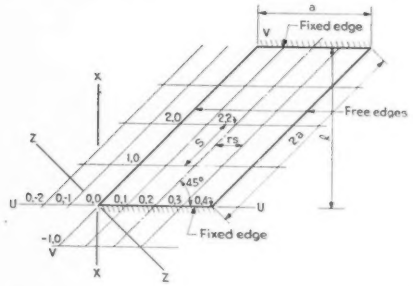


Fig. 7—Plan of skew plate

plate using skew coordinates is

$$\frac{\partial^4 w}{\partial v^4} - 4\sin\phi \frac{\partial^4 w}{\partial v^3 \partial u} + 2(2\sin^2\phi + 1)\frac{\partial^4 w}{\partial v^2 \partial u^2} - 4\sin\phi \frac{\partial^4 w}{\partial u^3 \partial v} + \frac{\partial^4 w}{\partial u^4} = p\frac{\cos^4\phi}{EI} \quad \text{(21)}$$

From previous considerations the finite difference equivalents at a typical point (Fig. 7) is given by

$$\frac{\partial^4 w}{\partial v^4} \doteq (w_{m+2,\,n} - 4w_{m+1,\,n} + 6w_{m,\,n} - 4w_{m-1,\,n} + w_{m-2,\,n}) \div s^4 \quad \text{(22a)}$$

$$\frac{\partial^4 w}{\partial u\, \partial v^3} \doteq (w_{m+2,\,n+1} - 2w_{m+1,\,n+1} + 2w_{m-1,\,n+1} - w_{m-2,\,n+1} - w_{m+2,\,n-1} +$$
$$2w_{m+1,\,n-1} - 2w_{m-1,\,n-1} + w_{m-2,\,n-1}) \div 4rs^4 \quad \text{(22b)}$$

$$\frac{\partial^4 w}{\partial u^2 \partial v^2} \doteq (w_{m+1,\,n+1} - 2w_{m,\,n+1} + w_{m-1,\,n+1} - 2w_{m+1,\,n} + 4w_{m,\,n} - 2w_{m-1,\,n} +$$
$$w_{m+1,\,n-1} - 2w_{m,\,n-1} + w_{m-1\,\,n-1}) \div r^2 s^4 \quad \text{(22c)}$$

$$\frac{\partial^4 w}{\partial u^3 \partial v} \doteq (w_{m+1,\,n+2} - 2w_{m+1,\,n+1} + 2w_{m+1,\,n-1} - w_{m+1,\,n-2} - w_{m-1,\,n+2} + 2w_{m-1,\,n+1} - 2w_{m-1,\,n-1} + w_{m-1,\,n-2}) \div 4r^3 s^4 \quad (22d)$$

$$\frac{\partial^4 w}{\partial u^4} \doteq (w_{m,\,n+2} - 4w_{m,\,n+1} + 6w_{m,\,n} - 4w_{m,\,n-1} + w_{m,\,n-2}) \div r^4 s^4 \quad (22e)$$

Letting $\phi = 45°$ and $r = \frac{1}{2}$ then the difference equation for the interior points is

$$1.414\,w_{m+2,\,n+1} + w_{m+2,\,n} + 1.414w_{m+2,\,n-1} - 5.657\,w_{m+1,\,n+2} + 30.14\,w_{m+1,\,n+1} - 36.00\,w_{m+1,\,n} + 1.858\,w_{m+1,\,n-1} + 5.657\,w_{m+1,\,n-2} + 16.00\,w_{m,\,n+2} - 96.00\,w_{m,\,n+1} + 166.0\,w_{m,\,n} - 96.00\,w_{m,\,n-1} + 16.00\,w_{m,\,n-2} + 5.657\,w_{m-1,\,n+2} + 1.858\,w_{m-1,\,n+1} - 36w_{m-1,\,n} + 30.14\,w_{m-1,\,n-1} - 5.657\,w_{m-1,\,n-2} + 1.414\,w_{m-2,\,n+1} + w_{m-2,\,n} - 1.414\,w_{m-2,\,n-1} = \frac{ps^4}{4EI} \quad (23)$$

This relationship is presented graphically in Fig. 8. Eq. (23) applies to all interior points.

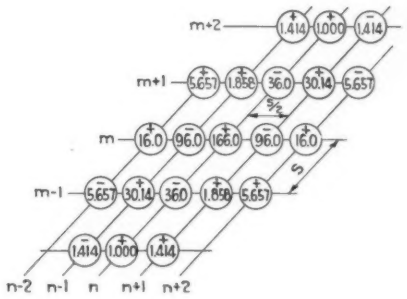


Fig. 8—Finite difference equation in skew coordinate system for interior points

At the fixed edge, $v = 0$, the displacement and rotation in the v direction has been set equal to zero.

Hence

$$w_{0,\,n} = 0 \quad (24a)$$

and

$$w_{1,\,n} = w_{-1,\,n} \quad (24b)$$

Along the free edges, it can be shown that the differential expressions necessary to make the moment and reaction normal to the edge equal to zero are

$$M_x = 0 = -\frac{EI}{\cos^2\phi}\left(\frac{\partial^2 w}{\partial u^2} - 2\sin\phi\,\frac{\partial^2 w}{\partial u\,\partial v} + \sin^2\phi\,\frac{\partial^2 w}{\partial v^2}\right) \quad (25a)$$

$$R_x = 0 = -\frac{EI}{\cos^3\phi}\left[\frac{\partial^3 w}{\partial u^3} - 3\sin\phi\,\frac{\partial^3 w}{\partial u^2 \partial v} + (2\cos^2\phi + 3\sin^2\phi)\,\frac{\partial^3 w}{\partial u \partial v^2} - (2\cos^2\phi\sin\phi + \sin^3\phi)\,\frac{\partial^3 w}{\partial v^3}\right] \quad (25b)$$

Expressing Eq. (25a) and (25b) in terms of finite differences, we have at $u = 0$ or $u = 4rs$

$$M_x = 0 \doteq \frac{w_{m,\,n+1} - 2w_{m,\,n} + w_{m,\,n-1}}{r^2 s^2} -$$

$$2\sin\phi\left(\frac{w_{m+1,\,n+1}-w_{m+1,\,n-1}-w_{m-1,\,n+1}+w_{m-1,\,n-1}}{4rs^2}\right)+$$

$$\sin^2\phi\left(\frac{w_{m+1,\,n}-2w_{m,\,n}+w_{m-1,\,n}}{s^2}\right) \quad \text{(26a)}$$

$$R_z=0\doteq\frac{w_{m,\,n+2}-2w_{m,\,n+1}+2w_{m,\,n-1}-w_{m,\,n-2}}{2r^3\,s^3}-3\sin\phi$$

$$\left(\frac{w_{m+1,\,n+1}-2w_{m+1,\,n}+w_{m+1,\,n-1}-w_{m-1,\,n+1}+2w_{m-1,\,n}-w_{m-1,\,n-1}}{2r^2\,s^3}\right)+$$

$$(2\cos^2\phi+3\sin^2\phi)$$

$$\left(\frac{w_{m+1,\,n+1}-2w_{m,\,n+1}+w_{m-1,\,n+1}-w_{m+1,\,n-1}+2w_{m,\,n-1}-w_{m-1,\,n-1}}{2r\,s^3}\right)-$$

$$(2\cos^2\phi\sin\phi+\sin^3\phi)\left(\frac{w_{m+2,\,n}-2w_{m+1,\,n}+2w_{m-1,\,n}-w_{m-2,\,n}}{2\,s^3}\right) \quad \text{(26b)}$$

When $r=\frac{1}{2}$ and $\phi=45°$, Eq. (26a) and (26b) reduce to

$$0=-0.707\,w_{m+1,\,n+1}+0.5\,w_{m+1,\,n}+0.707\,w_{m+1,\,n-1}+4.00\,w_{m,\,n+1}-9.00\,w_{m,\,n}+4.00\,w_{m,\,n-1}+0.707\,w_{m-1,\,n+1}+0.5\,w_{m-1,\,n}-0.707\,w_{m-1,\,n+1} \quad \text{(27a)}$$

$$0=-1.061\,w_{m+2,\,n}-3.485\,w_{m+1,\,n+1}+19.09\,w_{m+1,\,n}-13.49\,w_{m+1,\,n-1}+8.00\,w_{m,\,n+2}-26.00\,w_{m,\,n+1}+26.00\,w_{m,\,n-1}-8.00\,w_{m,\,n-2}+13.49\,w_{m-1,\,n+1}-19.09\,w_{m-1,\,n}+3.485\,w_{m-1,\,n-1}+1.061\,w_{m-2,\,n} \quad \text{(27b)}$$

which are presented graphically in Fig. 9. The coefficients at points $w_{m,\,-2}$ and

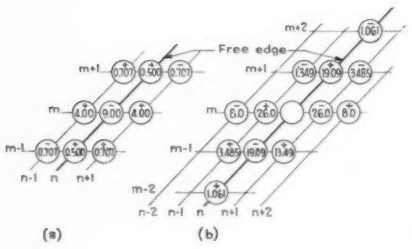


Fig. 9—Finite difference equations for moment and reaction at free edge.

$w_{m,\,-6}$ are eliminated by combining Eq. (27b) taken along the free edges at points w_{m-1}, w_m and w_{m+1} with Eq. (23) taken at point w_m. The resulting relationship is pictured in Fig. 10 for both edges. A sufficient number of equations can thus be written to evaluate the deflections at all points. These equations are solved in the same manner as shown in the preceding example. The bending moment in the v and x direction determined by the analysis are summarized in Table 7.

Crout's solution of simultaneous equations becomes unwieldly for a finer network than that used in the example. With more points, the number of operations to obtain the various coefficients is increased considerably. Therefore, it is preferable in those cases where a fine network is needed to use Southwell's relaxation method. N. M. Frocht in Chapter 8 of his *Photoelasticity*, Wiley, 1948, presents a concise introduction to the relaxation procedure.

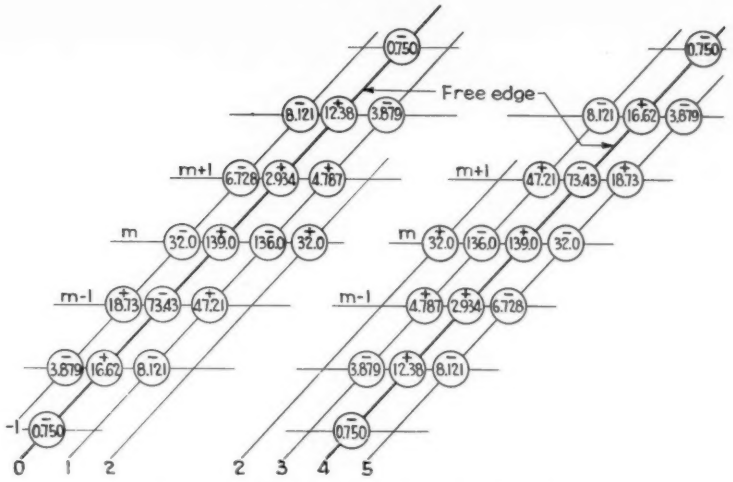


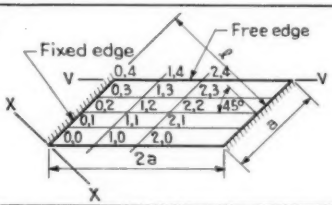
Fig. 10—Finite difference equations along free edges

TABLE 7—MOMENT IN SKEW PLATE, FIXED AT TWO EDGES, FREE AT OTHER TWO, SUBJECT TO UNIFORM LOAD

Δ = Deflection in terms of $\frac{wl^4}{EI}$

M_V = Moment parallel to free edges in terms of wl^2

M_X = Moment perpendicular to fixed edge in terms of wl^2



Point	0,0	0,1	0,2	0,3	0,4	1,0	1,1	1,2	1,3	1,4	2,0	2,1	2,2
Δ	0.0	0.0	0.0	0.0	0.0	.00233	.00262	.00299	.00358	.00456	.00489	.00476	.00469
M_V	-.0373	-.0420	-.0478	-.0573	-.0730	-.0018	-.0039	+.0103	+.0192	+.0338	+.0231	+.0265	+.0271
M_X	-.0747	-.0840	-.0957	-.1146	-.1459	-.0031	+.0031	+.0134	+.0285	+.0512	+.0170	+.0256	+.0281

SELECTED BIBLIOGRAPHY

1. Sheppard, W. F., "Central Differences Formulae," *Proceedings*, London Mathematical Society, V. 31, 1899.

2. Richardson, L. F., "The Approximate Arithmetical Solution by Finite Differences of Physical Problems Involving Differential Equations with an Application to the Stresses in a Masonry Dam," *Philosophy*, Trans. 210A, 1910.

3. Marcus, H., *Die Theorie Elasticher Gewebe und ihre Anwandung auf die Berechnung biesamer Platten*, Julius Spring, Berlin, Germany, 1924.

4. Holl, D. L., "Difference Methods and the Superposition Principle," *Journal of Applied Mechanics*, A.S.M.E., V. 3, No. 3, Sept. 1936.

5. Timoshenko, S., *Theory of Plates and Shells*, McGraw-Hill Book Co., Inc., New York, N. Y., 1940.

6. Southwell, R. V., *Relaxation Methods in Engineering Science*, University Press, Oxford, England, 1940.

7. Crout, P. D., "A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," *Transactions*, A.I.E.E., V. 60, 1941.

8. Jensen, V. P., "Analyses of Skew Slabs," *Bulletin*, No. 332, University of Illinois Engineering Experiment Station, Urbana, Ill., 1941.

9. Favre, H. "The Calculation of Oblique Slabs by the Method of Differences," *Trans.*, International Assn. for Bridge and Structural Engineering, V. 7, 1943-44.

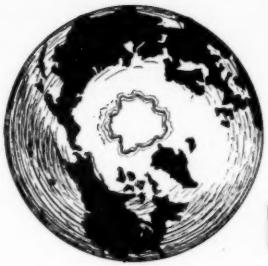
10. Newmark, N. M., "Numerical Procedures for Computing Deflections, Moments and Buckling Loads," *Transactions*, A.S.C.E., V. 108, 1943.

11. Southwell, R. V., *Relaxation Methods in Theoretical Physics*, University Press, Oxford, England, 1946.

12. Fox, L., "Some Improvements in the Use of Relaxation Methods for the Solution of Ordinary and Partial Differential Equations," *Proceedings*, Royal Society, A-190, 1947.

13. Salvadori, M. G., "Numerical Computations of Buckling Loads by Finite Differences," *Proceedings*, A.S.C.E., Dec. 1949.

14. Grinter, L. E., *Numerical Methods of Analysis in Engineering*, MacMillan Co., New York, N. Y., 1949.



Letters from Readers*

BY WAY OF SYNOPSIS

HOMER M. HADLEY questions the use of the plastic theory for the design of concrete flexural members.

LESLIE POOLE describes a unique design for a shopping center using lightweight concrete frames.

A READER chuckles over the etymological offspring of the word *concrete*.

Skepticism Regarding the Plastic Theory of Design (LR 47-66)

At the recent convention in Seattle of the American Society for Engineering Education a speaker advised his audience that the tenacity with which they clung to the teaching of the straight line theory (of flexure) for the design of reinforced concrete was "very much like continuing to teach geography students that the earth is flat when we know better." He added, "Perhaps I should not be quite so dogmatic because a transition period probably should be allowed between the teaching of even a known untruth and that which has been recognized for half a century or more and proved by research over and over again." The egregious error to which the speaker referred is the failure to teach "the plastic theory of design." "Today we are in the anomalous position of recognizing the plastic characteristics of concrete in the design of columns while adhering to the elastic theory in the design of flexural members."

Proof is needed

Without knowing the precise reasons why the plastic theory of design is not widely taught in the engineering colleges of the country, it occurs to the writer that inadequacy of supporting data and evidence constitutes a sound and valid reason for not teaching it. When in 1522 Magellan's expedition arrived at its starting point after continuously sailing in a westerly direction from that starting point for about three years, the feeling was general and widespread that the earth must be round. Subsequent to that time the experiment has been repeated again and again and always with identical results. Sail away—or fly away—from your starting point in any direction and continue long enough upon that course and presently you'll be back at your starting point. The earth *is* round. To quote again from the speaker: "as abundantly shown by research."

It was likewise in Magellan's time that mankind was coming to a wider realization of the fact that natural laws are persistent, inviolate and uniform throughout the world and are not created either by revelation, proclamation, authority or plain ballyhoo. A natural phenomenon observed, say in Berlin, Germany, or in Chicago, Ill., can

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under like conditions similarly be observed in Seattle, Wash., or in Denver, Colo., or anywhere else in the world and if the phenomenon cannot be duplicated and reproduced elsewhere then certainly there is no need for haste in accepting theories based upon the first reported findings—no need for haste in ascribing to such theories truth akin to that abiding in the fact that the earth is round. On the contrary there is every need to go slow, to find out exactly what is what before teaching or recommending the teaching of questionable theory to trusting students.

Concrete characteristics

The fact that concrete is brittle—that under blows it will crack and shatter, that unlike edges of steel which bend and buckle, concrete's thin edges and corners break completely off—in short, that concrete is lacking in tensile cohesiveness has long seemed to the writer to be one of its salient characteristics. It is because of its low tensile strength that reinforcement is combined with it to make the material of today's vast usage—reinforced concrete. But in either plain or reinforced form the brittleness, the insignificant tensile strength, persists and is an inherent characteristic of concrete.

Another salient characteristic of concrete is its heterogenous structure. Look at any specimen of cut or broken concrete that reveals its physical nature and what does one see? An utterly random, heterogeneous arrangement of aggregate particles, large and small, gripped and held in a matrix of hardened mortar and cement paste. That concrete's greatest strength should be in compression and that it should be weak in tension is certainly indicated by its structure.

Still another characteristic of concrete, which it shares with all other materials, is its unfailing tendency when subjected to axial compression to bow laterally. It seeks to relieve itself of load or to transfer loads to other elements by shortening with sidewise movement. To what degree this action is developed depends upon the loading, shape, dimensions, etc. but always it is present. With all materials this tendency to bow laterally involves curvature either of the entire specimen or of its parts. Likewise curvature involves bending and tensile stresses. With concrete this is a serious deficiency.

A realistic method?

Such being the basic and fundamental conditions respecting concrete, it was with amazement and incredulity that the writer read the paper "Plastic Theory of Reinforced Concrete Design" by Charles S. Whitney (*Trans.*, A.S.C.E., V. 107, 1942, p. 251). In the paper's synopsis it is stated to be "a realistic method" and one "which recognizes the true characteristics of the material determined by research."

This research appears to have consisted wholly of the work of two seniors at the University of Wisconsin and was presented in Mr. Whitney's Fig. 1, being titled "Typical Stress-Strain Curves for Concrete Cylinders in Compression." They show customary straight-line stress-strain relationships up to about 3200 psi at a unit strain of 0.0010, rounding off at a maximum stress 3500 to 4200 psi at unit strains of 0.0016 to 0.0020, and then gradually losing stress, more sharply at first, more slowly afterwards but at a unit stress of 0.0080 having a strength of 1500 psi. The continuity of the curves and the absence of any qualifying notes left the reader no other possible interpretation of them than that they presented the record of the cylinders' behavior and that the concrete was as sound and unbroken at the end of the tests as it was at the outset. It was from these cylinders that Mr. Whitney developed his proposed pattern of stress-distribution in flexure at ultimate load: a rectangular pattern with stress arbitrarily taken at $0.85f'_c$ and of constant value from the outermost fiber to the neutral axis where it abruptly terminated and vanished. The writer wrote some discussion of this paper and raised several questions regarding the tests but Mr. Whitney in his closure failed to answer them.

A year or two later Vernon P. Jensen, then Research Associate Professor of Theoretical and Applied Mechanics, University of Illinois, developed and published his paper "Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete." It appears in *Bulletin Series No. 345* of the Engineering Experiment Station, University of Illinois, and in the ACI JOURNAL, June 1943, *Proc.* V. 39, p. 565. Its pattern of stress distribution is essentially the same as Mr. Whitney's but is refined to avoid the abrupt change from full stress to no stress

at the neutral axis and it employs the full cylinder strength as maximum stress at ultimate load rather than Mr. Whitney's $0.85f'_c$.

The truly unique feature of Professor Jensen's paper is this, however: it reports physical data and evidence for only the minor part of its stress-pattern, *i.e.*, for the straight-line and largely inconsequential part adjacent to the neutral axis. For the major part of the stress pattern no physical evidence or data whatsoever is submitted—merely the speculation of Saliger as to what *may* be the "stress-strain diagram for concrete at strains approaching rupture." This reported speculation shows a stress-strain curve sweeping up to a maximum f'_c value at a unit strain of ϵ_o, then extending predominantly horizontally but with some loss in stress to a maximum strain of from 3 to 7 ϵ_o. In conjunction with the Saliger speculation Professor Jensen employs a new set of values for the modular ratio, n, which are at marked variance with those commonly employed on Joint Committee and ACI recommendations and writes—purely arbitrarily—an adaptation of the Saliger speculation into a cunning formula for β, which is the supposed measure of "the horizontal portion of the idealized stress strain curve." This formula tells us that with 2000-lb concrete the maximum stress holds unchanged until deformation under it becomes 4 times the deformation developed in going from a condition of no stress to maximum f'_c stress whereas with 4000-lb concrete it is merely equal to that deformation and with 6000-lb concrete it is but 1/3 of that deformation.

Discrete

Now the writer is filled with incredulity and disbelief when Professor Saliger or anyone else invites him to believe that plain concrete—*i.e.*, plain, ordinary, brittle concrete—can be subjected to test cylinder compression up to its maximum strength and that it will thereafter hold itself intact and proceed to deform like a metal under sustained maximum or reducing load.

Such disbelief led to the testing of a dozen cylinders a few years ago—see "When Concrete Becomes Discrete," *Civil Engineering*, Apr. 1950 and correction, May 1950. As might have been expected nothing remotely

Fig. 1—Cylinder at outer end of the "plastic range."

resembling the Saliger speculation or the Jensen horizontal β value was to be found in their stress-strain curves. There was most excellent agreement with the Whitney curves with these exceptions: it was found that at the extreme end of the curves the cylinders were completely shattered, broken and were held together merely by friction and aggregate interlock (Fig. 1); it was also found that at any point beyond the maximum the sudden application of additional load caused the cylinder to collapse utterly and break into countless fragments. It was therefore concluded that all portions of the curve beyond the maximum point were devoid of structural significance and that such worthless, fissured concrete might well be called "Discrete."

Plastic characteristics

There may still remain troubling thoughts in some minds over the agreement between beam tests and these plastic theories. How can the theories be unsound when this agreement exists? The writer is ignorant regarding the details of their development. Perhaps the Van den Broek theory of limit design may have influenced them. He would conjecture that the Jensen theory developed from the problem of harmonizing beam tests with a flexural pattern whose maximum stress was to be the 6 x 12 cylinder stress, f'_c. If this should be the case, he can but admire the mathematical nicety and crafts-

manship it reveals. Unfortunately, however, it doesn't fit the material, concrete, and it is not to be applied to it by mere assumption or by authority.

Actually the old and familiar straight-line theory of stress distribution, employing the Slater-Lyse constants, will agree with beam tests as well as do the plastic or ultimate theories. All that is needed to restore widespread satisfaction with it—if indeed that satisfaction has been lost by anyone as a result of the sustained efforts that have been made to discredit the straight-line theory—is a realization that the maximum flexural strength of concrete is not its 6 x 12 cylinder strength but is something much greater: 1.75 times the cylinder strength with 2000-lb concrete; 1.50 times that strength with 3000-lb concrete; 1.40 times that strength with 6000-lb concrete. Perhaps someone can devise a formula that will yield these values when the various concrete strengths, f'_c, are introduced into it. By calling the value ρ or ω, or something of the sort, a tone of scientific research or whatnot can be imparted to it. But by using these constants with f'_c the straight-line theory will yield perfect agreement with beam tests and since all portions of the cylinder stress-strain curves beyond the maximum are devoid of structural value, it is with the straight-line portions of such curves—the so-called "elastic range"—that we must perforce be content.

As for "recognizing the plastic characteristics of concrete in the design of columns while adhering to the elastic theory in the design of flexural members" there is only this to be said: "Don't let it worry you." The "plastic characteristics" observed in columns are the slow, long-time deformations which unload the concrete and load up the steel. The column formulas which are used resulted from the long-time column tests at Lehigh University and the University of Illinois and are in accordance with the results of those tests. But these column plastic characteristics are wholly different from the nonexistent, sudden ones that ultimate design has sought to create and find in concrete cylinders. The latter simply do not exist. They are pure fiction. There is no fiction, however, about the fact that the plastic characteristics found in columns likewise exist in beams. See Sergev's paper "Deformations of Steel Reinforcement During and After Construction," *Trans.* A.S.C.E., 1934, p. 1343, for an interesting account of stress conditions in beams and columns of a concrete building for the first 2¼ years of its existence. Fortunately, however, in the ordinary case of short spans and fair depths they are not disturbing in their deformational aspects. But with long spans and shallow depths they are no joke. Fig. 2 illustrates this fact in the case of an otherwise fine bridge wherein the camber allowances were wholly inadequate.

Fig. 2—View of deck of shallow box-girder bridge, inadequately cambered, whose spans of 72 ft, 73 ft and 100 ft sagged ½ in., 1½ in. and 2½ in., respectively, in about 2½ years

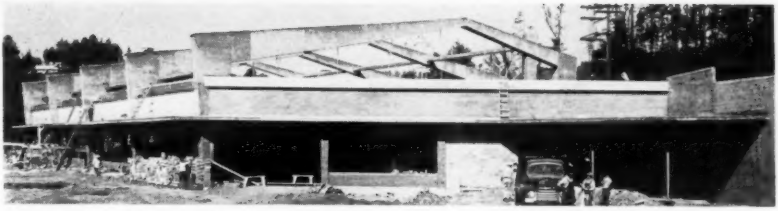
The writer is of the opinion that the earth is round and that the "plastic theory of design" for concrete flexural members is or should be as flat as a punctured balloon.

HOMER M. HADLEY, consulting engineer, Seattle, Wash.

Roof Suspended from Lightweight Concrete Frames (LR 47-67)

Interesting architectural design and unique engineering techniques—which feature the use of concrete frames exposed above the building's roof—characterize a new shopping center in Portland, Ore. The roof and ceiling of the new building are suspended from five lightweight concrete frames by steel rods (Fig. 1), enabling the construction of 40,000 sq ft of store floor space without pillars.

The Fred Meyer Burlingame Shopping Center presented a rather complex design problem. As is true with most buildings of this kind, low cost of construction was of primary importance except as it might be



Courtesy Ackroyd Photography, Inc.

Fig. 1—The concrete frames eliminated columns in the store area and also presented a pleasing architectural treatment.

affected by insurance rates and cost of maintenance. An open floor space free from columns or other obstructions was desired since it would allow complete freedom in the arrangement of fixtures and merchandise. It was also important that the building be attractive to the eye.

The major factor affecting the design was the nature of the site. The shape of the property and the zone limits established the shape of the building as shown in Fig. 2. The building is located at the foot of a hill and the design had to be one that would stand out and still be in harmony with the background.

The use of beams and joists was immediately discarded as necessitating too many columns within the shopping area. The use of trusses of any conventional type would present a large, unattractive roof area to view and would involve some rather complicated and expensive framing. It was finally decided that some type of arch projecting above the roof would more nearly fulfill all requirements than any other type of construction.

Laminated wood as well as steel arches were both discarded as being too costly and too expensive to maintain. Concrete seemed the most practical material from any point of view.

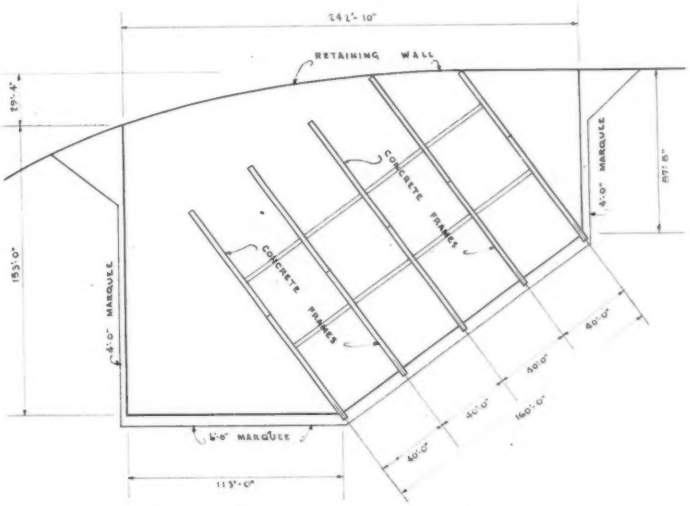


Fig. 2—Shape of the shopping center as established by the property

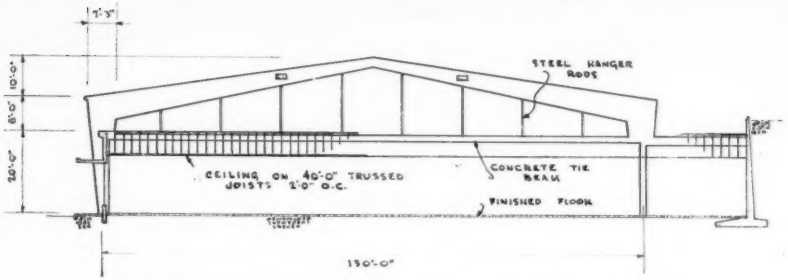


Fig. 3—The frames are neither a true arch nor a truss, but a combination

The design selected (Fig. 3), is neither a true arch nor a truss but might be considered a combination of the two. Diagonal web members are eliminated, the top chord being designed to resist the bending induced by the roof load. The roof load is carried directly on the lower chord which is in turn supported from the upper chord by steel rods.

The selection of this design resulted in economies in roof framing which resulted in an over-all cost comparable to that of any other type of construction for a building of the same class. It must be remembered that the shape of the building would make any method of roof framing rather expensive.

Roof and ceiling joists were laced together to form light parallel chord trusses spanning the 40 ft between the concrete frames. These trussed joists were prefabricated then hung from the lower chords of the frames at 2 ft centers. This reduced the cost of framing the ceiling and effectively speeded completion of the building.

To avoid making the frames too massive in appearance it was decided to use a concrete mix developing a compressive strength of 3000 psi in 28 days. Using concrete of this strength made it possible to hold the size of the section within the desired limits. The amount of reinforcement required, however, was so great that it was feared some difficulty might be encountered in placing the concrete without the formation of rock pockets. This was particularly important in the bottom chord where complete embedment was necessary to fully develop the steel in tension. This led to the consideration of lightweight concrete as a suitable material.

Preliminary investigation revealed that most of the lightweight aggregates available on the local market would not develop the required strength without the addition of a considerable amount of sand which added too much weight to the concrete. Lite Rock, an expanded shale aggregate, was chosen as having the lightest weight for the required strength of any of the local materials.

The maximum weight of concrete allowed by the specifications was 95 lb per cu ft. This resulted in reducing the weight of each complete frame from 115 tons to 73 tons. Upon investigation it was discovered that this reduction in dead load reduced the required amount of reinforcing steel enough to justify the additional $5.00 per yd spent for the lightweight concrete. This smaller amount of steel and the ease of placing the lightweight concrete resulted in the contractor being able to place the concrete without noticeable honeycombing.

As the job progressed laboratory tests proved original assumptions to be conservative. While the dry weight varied between 90 and 95 lb per cu ft, the 28-day strength varied from 3640 to 6580 psi. This variation in strength is explained in part by the fact that we did some experimenting during the progress of the job. We varied the water-cement ratio, changed the grading of the aggregate and changed the size of the batch in the mixer. We found that the addition of a small amount of quarry dust and reducing the size of the batch in the mixer were important factors in increasing the strength of the concrete.

The design of this building again demonstrated that there is a direct relationship

between good engineering and good architecture. We achieved the desired architectural effect and in the final analysis the cost of construction showed no increase.

LESLIE POOLE, engineer,
Portland, Ore.

Concrete's Etymological Offspring (LR 47-68)

(The writer of a recent letter to the editor was having a laugh about the etymological offspring of the word "concrete," a chuckle we couldn't help but pass on to JOURNAL *readers. He writes in very good humor as follows—*EDITOR)

The word *concrete* has spawned a large crop of etymological offspring, some of which are of highly dubious legitimacy. The following have been noted, in print, in reputable technical journals.

Brikcrete	*Patchcrete*
Calcrete	*Plasticrete*
Celocrete	*Pumicrete*
Glascrete	*Pumpcrete*
Jetcrete	*Sakcrete*
Joltcrete	*Screwcrete*
Lenscrete	*Shadecrete*
Mascrete	*Strestcrete*

I do not know if this list will be of interest to you; I send it because it amused me.

READER

Current Reviews*

of Significant Contributions in Foreign and Domestic Publications

Foamed slag

Building Industry Distributors Course B, No. 12, Foamed Slag Producers Federation Ltd., London, 1948, 16 pp.

BUILDING SCIENCE ABSTRACTS Aug. 1949

The manufacture and properties of foamed slag aggregate and of foamed slag concrete are described, and the principal uses of foamed slag concrete are briefly indicated.

Reinforced concrete sections subject to unsymmetrical bending

K. HRUBAN, *Technicky Obzor* (Prague), V. 58, 1950, No. 8, pp. 113-117

Reviewed by IVAN M. VIEST

This paper deals with reinforced concrete beams of rectangular cross section subjected to unsymmetrical bending and combination of unsymmetrical bending with direct compression or tension. Equations are derived for factor of safety against failure on the basis of an ultimate theory assuming a rectangular stress block. The theory is supplemented by four numerical examples.

Prefabrication of concrete members (La prefabrication des materiaux en beton)

Revue des Materiaux (Paris), No. 418, July 1950, pp. 243-246

Reviewed by PHILLIP L. MELVILLE

To speed reconstruction the French national railroads have used precast members successfully in repair shops and round houses. Casting was done in steel-lined wooden molds or later in unlined concrete ones. Aggregate gradation was carefully controlled and the water-cement ratio kept as low as possible. V-shaped columns, window frames, slabs are among those described.

Design and calculation of flat slab floors (Ontwerp en berekening van paddestoelvloeren)

A. M. HAAS, Martinus Nijhoff, 's Gravenhage, 245 pp. 13 guilders

Reviewed by J. W. T. VAN ERP

The author develops along purely analytical lines a complete mathematical investigation of flat slab structures. Due attention is paid to the monolithic character of this type of construction and to the fact that the floor thickness is increased in and around the column head. This last point has, in former theoretical investigations, been insufficiently considered. For this reason the results presented as design data should prove valuable, the more so because a summary in English is included.

The design of continuous beams (Zur Berechnung der Durchlauftraeger)

R. LAMBERG, *Die Bautechnik* (Berlin), July 1949, No. 7, pp. 205-208

Reviewed by RUDOLPH FISCHL

The method of the conjugated matrix for the design of continuous beams deserves to be more generally known. It is especially useful in solving a sequence of loading conditions.

The system of three-moment equations can be written in matrix form. By solving these equations for a unit load, a conjugated matrix can be formed. Using this matrix and the real loading members, each moment

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equation can be written down immediately. The factors of these equations are built after certain laws, which are developed in this paper. General formulas and rules are given, after which the moment factors can be figured quite mechanically.

Formulas are given for beams with four to six supports, but can easily be extended to more supports. Two examples and a table with loading members are given.

Influences on concrete and reinforced concrete (Einfluesse auf Beton und Stahlbeton)

A. KLEINLOGEL, Wilhelm Ernst & Sohn, Berlin, 1950, 5th Edition, 340 pp. 24.50 DM plus postage

Reviewed by RUDOLPH FISCHL

The fifth edition of this well-known reference book for practical use has been completely revised and enlarged.

Chemical, mechanical, and other actions on concrete and reinforced concrete are listed in alphabetic order. It is shown how they effect the proper qualities of concrete, and means of protection are discussed.

Numerous references are given, and an appendix translates chemical terms into English and French.

Statical diary (Statisches tagebuch)

KARL STADOR, Karl Marklein, Dusseldorf, 1950, 200 pp. $3.40

Reviewed by WERNER H. GUMPERTZ

The author attempts to give the young engineer some structural information in widely scattered fields. The book presents elementary structural knowledge which will be especially useful for the design of simple structures and temporary construction. It will probably be most useful to the field engineer whose structural knowledge is limited. The author has confined himself to the realistic treatment of essential problems which are likely to come up on any major construction job.

This book fulfills a need in a field where theoretical refinement is undesirable.

Thin shell roof construction (Schaaldaken)

W. VAN DER SCHRIER, *Cement* (Amsterdam), No. 11-12, 1949, p. 220

Reviewed by J. W. T. VAN ERP

The necessity for utmost economy has pushed the development of thin shell roof construction. Its advantages for industrial buildings are mainly:

1. Small amount of material per sq ft of roof area. For this structure, where the shells are 24 ft wide and span 45 ft there is 4.9 psf steel or 0.6 cu ft per sq ft of concrete.
2. Large spans are possible with a minimum number of columns.
3. Favorably diffused daylighting is obtained in sawtooth-roof construction by the reflection of the light against the concave underside of the roofshells.
4. Accelerated construction is possible due to the use of uniform units. Formwork is light, a factor of importance where construction timber is scarce.

The manufacturing area described had quarter circle sawtooth-roof sections stiffened by its adjacent gutter section. The shell is 3¼ in. thick in the middle. Concrete mix was 1 : 1½ : 2½. Two and one-half in. vermiculite insulates the top of shells.

Strontium and barium cements

AL. BRONSKI, *Cement and Lime Manufacture* (London), V. 23, No. 2, 1950, p. 30

Reviewed by R. P. VELLINES

Some interesting properties of cements made from the alkaline metallic minerals other than calcium are described. Reference is made to original investigation reported in the proceedings of the Rumanian Chemical Society (*Bulletin of Pure and Applied Chemistry*, V. 37, 1934) and to later work by W. F. Shurawlew (1940) of Leningrad and G. Gallo (1947) in Italy. Barium cement is said to have an extraordinary resistance to chemical attack in general and nearly absolute resistance to corrosive action of sea water. Protection against X-rays and gamma-rays is indicated. Strontium cement is somewhat less resistant generally than the barium cement. Simple, double and triple compounds of which alkaline cements are constituted are indicated.

The metallic alkalies (lithium, sodium and potassium) and the elements (magnesium, zinc and lead) are incapable of forming hydraulic cements. Theoretical and practical research on barium, strontium, barium-strontium and calcium-barium cements may lead to solution of problems associated with the constitution of cement, the heat of hydration and other matters not at present explicable, such as setting, hardening, shrinkage, expansion and strength.

The Sclayn bridge over the Meuse (Continued) (Le pont de Sclayn sur le Meuse (Suite et fin))

ERWIN STORRER, *Annales des Travaux Publics de Belgique*, No. 4, Aug. 1950
Reviewed by CHARLES C. ZOLLMAN

The author deals first with the principles of design on which this continuous Blaton-Magnel prestressed structure is based. He then describes some phases of the construction; falsework and formwork, concreting schedule for the various parts, assembly of wire units and their installation and stressing. A short conclusion discusses the deformation of the continuous deck.

The outstanding feature of this paper is the description of the vertical transverse crack which occurred during construction at about 10 ft from the centerline of the river pile. This crack progressively increased, within a week, to a final width of about ¼ in. at the top of the box-girder and remained almost imperceptible at the bottom. The bridge was literally broken in two parts, one part having rotated about a generatrix of the soffit.

Its causes were the tensile stresses set up in the concrete in one of the spans before prestressing and were due to (1) settlement of the falsework because of dehydration and compression of the wood in the temporary wooden pile bent supporting the steel girders, (2) deflection of the steel girders carrying the formwork above the channel opening and compression of the wooden wedges placed in between and (3) restraint of the shrinkage of the top slab.

The author elaborates on what, thanks to prestressing, was a harmless crack which gradually closed almost completely as the prestressing progressed. In his conclusion he points out the perfect behavior of this impressive structure when it was tested for static and dynamic loads before its dedication earlier this year.

Elementary stress analysis

PHILLIP E. SONESON, Pitman Publishing Co., New York, 1949, 351 pp. $5.00
Reviewed by A. A. TOPRACTSOGLOU

This attractive and well organized book is an excellent textbook for a one semester course in determinate structures. The sequence followed and the clear presentation of the material makes the subjects easy to follow. Throughout the book the fundamental principles of statics are emphasized as they are applied in the analysis of bridge and building structures.

A great number of illustrative examples are given in each chapter with part of the theory being explained in these examples. Numerous problems, similar to those met in practice, are given at the end of each chapter for assignment to students. In general the book lends itself readily to teaching.

In the first three chapters a thorough review is given on forces, reactions, and moment and shear diagrams. Several analytical and a graphical method for various types of trusses are presented in Chapter IV.

Wind forces are treated in an excellent detailed way. The material covered is more than what is needed for a course on determinate structures. The wind pressures on different types of roofs and other structures are analyzed. References on wind forces are listed at the end of the chapter.

The treatment of mill buildings is very satisfactory. Although the graphics given is sufficient it has been spread out in the various chapters thus making it somewhat difficult for the beginner to grasp once and for all this important subject.

Highway bridges follow an introductory chapter on moving loads and influence lines. Railway bridges are treated next. At the end of the last two chapters the author gives the latest specifications. The book ends with a short chapter on deflection of trusses.

Reinforced concrete construction (Der Stahlbetonbau)

RUDOLPH SALIGER, F. Deuticke, Vienna, 1949, 7th edition, 644 pp. $15.18
Reviewed by RUDOLPH FISCHL

This standard textbook by the internationally known Professor Saliger of the Technical University in Vienna, has kept pace since its first appearance with the newest development in reinforced concrete construction.

During the 16 years which have passed since the previous edition, many new ways have been found which made it necessary to rewrite most of the chapters of the book.

Special attention has been paid to the results of modern research such as the shape of the aggregates, water-cement ratio, creeping, buckling and flexibility of concrete, steel with high yield point, prestressed reinforcing

and other topics. A new theory, covering the appearance of cracks in tension members and beams, is presented. The theory of plastic range is developed and its practical application is shown.

The extensive subject is divided into four parts. The first part treats the material. Starting with the history of reinforced concrete, it discusses further the manufacture, properties, strength and deformation of the material, the reinforcement, basic shapes, safety against rust and fireproofing.

The second part deals with axial stresses, bending within elastic limits, resistance of the tension and compression zone, dimensioning for bending, eccentric compression and tension forces, shear resistance, bond and torsion.

The third part treats continuous beams, two-way slabs, flat slabs, frames, Vierendeel trusses, bins, arches, cupolas, thin shells and other constructions.

The fourth part contains specifications and numerous references. 700 figures and 140 tables supporting the text make this an outstanding textbook for students as well as an up-to-date reference book for the practicing engineer.

It may be mentioned that the publisher did an excellent job, too. Paper, print and binding are of the best quality, thus giving the book a distinguished appearance.

Distribution of deformation (A new method of structural analysis)

C. V. Kloucek, Prague, 1949, 510 pp.
Reviewed by Douglas M. Henry

This work, which is apparently available in the English translation only in a limited edition, is a valuable contribution to engineering literature. Although the translation is above average in clarity, the term "deformation" is perhaps not too well chosen; it refers to angular rotation of joints. The concept of distribution of angular rotations, based upon slope-deflection equations, is by no means new in American literature. However, it appears that Mr. Kloucek has given more thought than has any other exponent of the method toward converting a rather obvious but somewhat unwieldly concept into an equally obvious but reasonably workable computation scheme. It seems rather strange to the reviewer that structural analysis methods which are based upon displacements or deformations (rotations) have not "caught on" when they have been placed in competition with methods which deal with moments or with certain functions of the stress components, since displacement relationships usually involve fewer unknowns and are often easier to visualize. The translator states that the method is being taught in a number of European universities, and that several European specialists (for example, Professor Kleinlogel) consider it the quickest known solution for complex problems.

The author's method is not iterative, nor does it involve the solution of simultaneous equations (except in certain incidental features); rather, the effect of releasing successive joints is represented by a continuous fraction, and the computer may evaluate the complete fraction or he may interrupt it after one or more terms, depending upon the structural conditions and the degree of accuracy required.

In Part I of the 510-page work, cases are treated in which the joints are not displaced from their initial locations, but are subjected only to rotations. The treatment covers continuous beams, continuous frames, closed frames, and multi-story frames. The treatment is straight-forward and can be followed readily by any designer, even though it contains 654 numbered equations. Approximations which are introduced to simplify the computations follow directly from the generalized equations, and permit the user to select a degree of approximation which fits the requirements of the problem. Part II deals with cases in which joint displacements must be considered, and naturally it is somewhat more involved than Part I. The more complex of these cases may be simplified by concepts which appear to be new, and which may be of some value aside from their application in the deformation distribution method. Many examples are included of all phases which are treated by the author, including a fully worked example of a 25-story frame with sidesway.

The book is well printed and appears to contain few typographical errors. Although it seems to the reviewer that the method possesses considerable merit, it is doubtful that it will be widely accepted in this country, where other systems are so well established, unless our educators should see fit to include it in college curricula.